This booklet contains 24 printed pages

PAPER – 1 : PHYSICS, CHEMISTRY & MATHEMATICS

Do not open this Test Booklet until you are asked to do so.

Read carefully the instructions on the Back Cover of this Test Booklet.

A

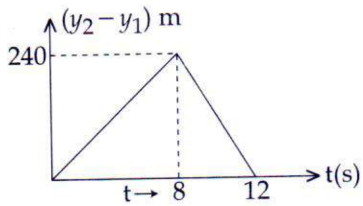
Important Instructions :

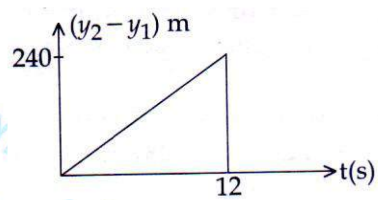
1. Immediately fill in the particulars on this page of the Test Booklet with Blue/Black Ball Point Pen. Use of pencil is strictly prohibited.
2. The Answer Sheet is kept inside this Test Booklet. When you are directed to open the Test Booklet, take out the Answer Sheet and fill in the particulars carefully.
3. The test is of 3 hours duration.
4. The Test Booklet consists of 90 questions. The maximum marks are 360.
5. There are three parts in the question paper A, B, C consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage. Each question is allotted 4 (four) marks for each correct response.
6. Candidates will be awarded marks as stated above in instructions No.5 for correct response of each question, ¼ (one fourth) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.
7. There is only one correct response for each question. Filling up more than one response in each question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 6 above.
8. Use Blue/Black Ball Point Pen only for writing particulars/marking responses on Side-1 and Side-2 of the Answer Sheet. Use of pencil is strictly prohibited.
9. No candidate is allowed to carry any textual material, printed or written, bits of papers, pager, mobile phone, ay electronic device, etc. except the Admit Card inside the examination room/hall.
10. On completion of the test, the candidate must hand over the Answer Sheet to the invigilator on duty in the Room/Hall. However, the candidates are allowed to take away this Test Booklet with them.
11. The CODE for this Booklet is A. Make sure that the CODE printed on Side – 2 of the Answer Sheet and also tally the serial number of the Test Booklet and Answer Sheet are the same as that on this booklet. In case of discrepancy, the candidate should immediately report the matter to the invigilator for replacement of both the Test Booklet and the Answer Sheet.
12. Do not fold or make any stray mark on the Answer Sheet.

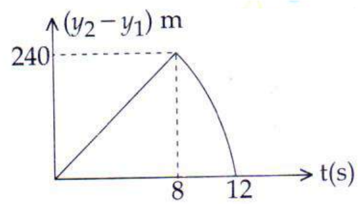
JEE-MAIN-2015 SOLUTIONS

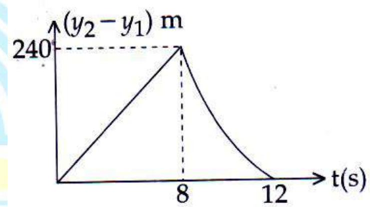
PHYSICS

1. Two stones are thrown up simultaneously form the edge of a cliff 240 m high with initial speed of 10 m/s and 40 m/s respectively. Which of the following graph best represents the time variation of relative position of the second stone with respect to the first (Assume stones do not rebound after hitting the ground and neglect air resistance, take g = 10 m/s^2)
The figure are schematic and not draw to scale)

(1) 

(2) 

(3) 

(4) 

Ans. (3)

Sol: $y_1 = 10t - 5t^2$

$y_2 = 40t - 5t^2$

And $y_2 - y_1 = 30\,t$ for $t \le 8$ s

At t = 8s, first stone is on the ground and the second stone is at the edge of the diff & moving down.

So at t = 8 s, $y_2 - y_1 = 240$ m

For $t > 8$ $\quad y_2 - y_1 = 240 - 40\,t - \frac{1}{2}\,g\,t^2$

Hence option (3)

2. The period of oscillation of simple pendulum is $T = 2\pi\sqrt{\frac{L}{g}}$. Measured value of L is 20.0 cm known to 1 mm accuracy and time for 100 oscillations of the pendulum is found to be 90 s using wrist watch of 1 s resolution. The accuracy in the determination of g is

(1) 2% (2) 3% (3) 1% (4) 5%

Ans. (2)

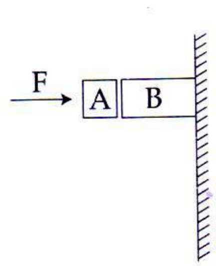
Sol: $g = \frac{4\pi^2 L}{T^2}$

$\frac{\Delta g}{g} = \frac{\Delta L}{L} + \frac{2\Delta T}{T}$

$= \frac{1}{200} + \frac{2\times 1}{90} = 2.72\% \approx 3\%$

$\therefore$ correct option (2)

3.

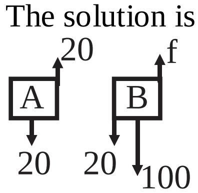


Given in the figure are two blocks A and B of weight 20 N and 100 N, respectively. These are being pressed against a wall by a force F as shown. If the coefficient of friction between the blocks is 0.1 and between block B and the wall is 0.15, the frictional force applied by the wall on block B is

(1) 100 N (2) 80 N (3) 120 N (4) 150 N

Ans. (3)

Sol. The solution is based on the assumption that the blocks are in equilibrium.



f = 120 N

$\therefore$ Correct option (3)

4. A particle of mass m moving in the x direction with speed 2v is hit by another particle of mass 2m moving in the y direction with speed v. If the collision is perfectly inelastic, the percentage loss in the energy during the collision is close to :

(1) 44% (2) 50% (3) 56% (4) 62%

Ans. (3)

Sol. From law of conversation of linear momentum

Centre of Mass(x-direction) $\Rightarrow m.2v = 3mv_x \Rightarrow v_x = \frac{2}{3}v$

(y-direction) $\Rightarrow 2m.v = 3mv_y \Rightarrow v_y = \frac{2}{3}v$

$$\therefore \text{ \% Loss} = \frac{\frac{1}{2}m(2v)^2 + \frac{1}{2}.2m(v)^2 - \frac{1}{2}(3m).\left[\frac{4}{9}v^2 + \frac{4}{9}v^2\right]}{\frac{1}{2}m(2m)^2 + \frac{1}{2}.2m(v)^2} = 56\%$$

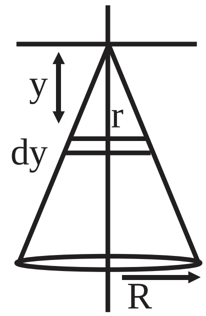
$\therefore$ correct option (3)

5. Distance the centre of mass of a solid uniform cone form its vertex is Z_0. If the radius of its base is R and its height is h then Z_0 is equal to:

THE NARAYANA GROUP

(1) $\frac{h^2}{4R}$ (2) $\frac{3h}{4}$ (3) $\frac{5h}{8}$ (4) $\frac{3h^2}{8R}$

Ans. (2)



Centre of mass $= \int_O^H \left(\frac{M}{\frac{1}{3}\pi R^2 H}\right) \times \frac{\pi r^2 y dy}{M}$

$= \frac{3}{R^2 H}\int_O^H r^2 y.dy = \frac{3}{R^2 H}\int \frac{R^2}{H^2} y^3.dy$

$= \frac{3}{4}\ H$

$\therefore$ correct option (2)

6. From a solid sphere of mass M and radius R cube of maximum possible volume is cut. Moment of inertia of cube about an axis passing through its centre and perpendicular to one of its faces is:

(1) $\frac{MR^2}{23\sqrt{2}\pi}$ (2) $\frac{MR^2}{16\sqrt{2}\pi}$ (3) $\frac{4MR^2}{9\sqrt{3}\pi}$ (4) $\frac{4MR^2}{3\sqrt{3}\pi}$

Ans. (3)

Sol. For maximum volume of cube, body diagonal of cube = Diameter of sphere

$a\sqrt{3} = 2R$ a = side of cube

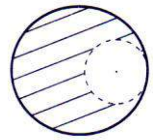
Volume of cube = V = $\left(\frac{2R}{\sqrt{3}}\right)^3$

Mass of cube $m = \frac{M}{\left(\frac{4}{3}\pi R^3\right)} \times \left(\frac{2R}{\sqrt{3}}\right)^3 = \frac{2M}{\pi\sqrt{3}}$

Moment of Inertia = $\frac{m}{6}(\text{side})^2 = \frac{4}{9\sqrt{3}}\frac{MR^2}{\pi}$

$\therefore$ correct option (3)

7. From a solid sphere of mass M and radius R, a spherical portion of radius $\frac{R}{2}$ is removed, as shown in the figure. Taking gravitational potential V = 0 at r = ∞, the potential at the centre of the cavity thus formed is: (G = gravitational constant)



(1) $\frac{-GM}{2R}$ (2) $\frac{-GM}{R}$ (3) $\frac{-2GM}{3R}$ (4) $\frac{-2GM}{R}$

Ans. (2)

Sol: Mass of complete sphere = M

"–ve" mass of cavity $M^1 = \frac{M}{8}$

Potential due to complete sphere $V_1 = -\frac{GM}{2R^3}\left(3R^2 - r^2\right)$

$\because r = \frac{R}{2}$ $\therefore v_1 = -\frac{11GM}{8R}$

Potential due to cavity $V_2 = \frac{-3GM^1}{2R^1} = -\frac{3G\left(\frac{M}{8}\right)}{R}$

$V_2 = -\frac{3GM}{8R}$

Now net potential at the given point

$V = V_1 - V_2 = \left(\frac{-11GM}{8R}\right) - \left(\frac{-3GM}{8R}\right)$

$\boxed{V = \frac{-GM}{R}}$

∴ Correct option (2)

8. A pendulum made of a uniform wire of cross sectional area A has time period T. When an additional mass M is added to its bob, the time period changes to T_M. If the Young's modulus of the material of the wire is Y then $\frac{1}{Y}$ is equal to :

(g = gravitational acceleration)

(1) $\left[\left(\frac{T_M}{T}\right)^2 - 1\right]\frac{A}{Mg}$ (2) $\left[\left(\frac{T_M}{T}\right)^2 - 1\right]\frac{Mg}{A}$

(3) $\left[1 - \left(\frac{T_M}{T}\right)^2\right]\frac{A}{Mg}$ (4) $\left[1 - \left(\frac{T}{T_M}\right)^2\right]\frac{A}{Mg}$

Ans. (1)

Sol. $T = 2\pi\sqrt{\frac{\ell}{g}}$ $\left\{\begin{array}{ll} \because & Y = \frac{Mg\ell}{A.\Delta\ell} \\ \therefore & \frac{\Delta\ell}{\ell} = \frac{mg}{A.y} \end{array}\right\}$

$T^2 \propto l$

$\frac{T_m^2}{T^2} = \frac{\ell(1+\Delta\ell)}{\ell} = 1+\Delta\ell$

$\frac{T_m^2}{T^2} = 1+\frac{Mg}{AY}$

$\left(\frac{T_m^2}{T^2}-1\right) = \frac{Mg}{AY}$

$$\boxed{\frac{1}{Y} = \left(\frac{T_m^2}{T^2}-1\right)\frac{A}{Mg}}$$

9. Consider a spherical shell of radius R at temperature T. The black body radiation inside it can be considered as an ideal gas of photons with internal energy per unit volume $u = \frac{U}{V} \propto T^4$ and pressure. $p = \frac{1}{3}\left(\frac{U}{V}\right)$. If the shell now undergoes an adiabatic expansion the relation between T and R is :

(1) $T \propto e^{-R}$ (2) $T \propto e^{-3R}$ (3) $T \propto \frac{1}{R}$ (4) $T \propto \frac{1}{R^3}$

Ans. (3)

Sol: $U \propto T^4 V$

$dU = T^4 dV + 4T^3 V.dT$

$dW = P.dV = \frac{U}{3V}dV = \frac{T^4}{3}dV$

$dQ = dW + dU = 0$

$\frac{T^4}{3}.dV + T^4.dV + 4T^3V.dT = 0$

$\frac{4}{3}\frac{dV}{V} + 4\frac{dT}{T} = 0$

$\frac{1}{3}\ln V + \ln T = \text{Constant}$

$TV^{1/3} = \text{Constant}$

$T \propto V^{-1/3}$

$T \propto R^{-1}$

10. A solid body of constant heat capacity 1 J/°C is being heated by keeping it in contact with reservoirs in two ways:

(i) Sequentially keeping in contact with 2 reservoirs such that each reservoir supplies same amount of heat.

(ii) Sequentially keeping in contact with 8 reservoirs such that each reservoir supplies same amount of heat.

In both the cases body is brought form initial temperature 100^oC to final temperature 200^oC. Entropy change of the body in the two cases respectively is

(1) ln2, 4ln2 (2) ln2, ln2 (3) ln2, 2ln2 (4) 2ln2, 8ln2

Ans. (No Answer)

Sol. $\Delta S = C\ln\frac{T_1}{T_i} + c\ln\frac{T_f}{T_1}$

$$\Delta s_1 = C\ln\frac{T_f}{T_i} = \ln\frac{473}{373} = \ln 1.27$$

$$\Delta s_2 = c\ln\frac{T_1}{T_i} + C\ln\frac{T_2}{T_1} + \ldots .c\ln\frac{T_f}{T_7}$$

$$= c\ln\frac{T_f}{T_i} = \ln\frac{473}{373} = \ln 1.27$$

None of the given options is correct.

11. Consider an ideal gas confined in an isolated closed chamber. As the gas undergoes an adiabatic expansion, the average time of collision between molecules increases as V^q, where V is the volume of the gas. The value of q is: $\left(\gamma = \frac{C_p}{C_v}\right)$

(1) $\frac{3\gamma+5}{6}$ (2) $\frac{3\gamma-5}{6}$ (3) $\frac{\gamma+1}{2}$ (4) $\frac{\gamma-1}{2}$

Ans. (3)

Sol: $t = \frac{\text{mean free path}(\lambda)}{v}$

$$\because\ \lambda \propto \frac{T}{P} \text{ and } v \propto \sqrt{T}$$

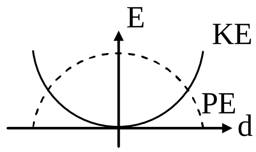
$$\therefore\ t \propto \frac{\sqrt{T}}{P}$$

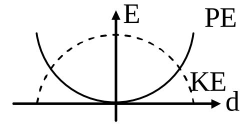
$$\propto T^{1/2}V^{\gamma}$$

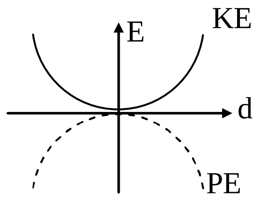
$$\propto V^{\frac{1+\gamma}{2}}$$

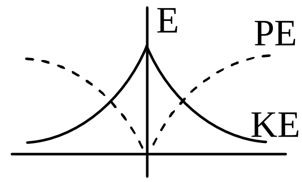
12. For a simple pendulum, a graph is plotted between its kinetic energy (KE) and potential energy (PE) against its displacement d. Which one of the following represents these correctly?

(graphs are schematic and not drawn to scale)

(1) 

(2) 

(3) 

(4) 

Ans. (2)

Sol: In SHM

$KE = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2(A^2 - d^2);$

$PE = \frac{1}{2}Kd^2 = \frac{1}{2}m\omega^2 d^2;$

$\therefore$ 'KE' Vs 'd' is a parabola opening downward

$\therefore$ 'PE' Vs 'd' is a parabola opening upward

$\therefore$ option (2) is correct.

13. A train is moving on a straight track with speed 20 ms^{-1}. It is blowing its whistle at the frequency of 1000 Hz. The percentage change in the frequency heard by a person standing near the track as the train passes him is (speed of sound = 320 ms^{-1}) close to:

(1) 6 % (2) 12 % (3) 18 % (4) 24 %

Ans. (2)

Sol: When the train is approaching $f_a = f_0\left[\frac{V}{V - V_s}\right]$

When the train crosses the person $f_a' = \frac{f_0 V}{V + V_s}$

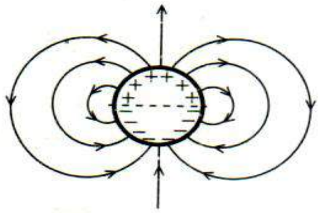
$\therefore$ % change in frequency = $\frac{\Delta f}{f_a} \times 100 = \frac{f_a - f_a'}{f_a} \times 100$

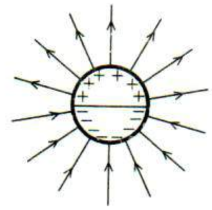
$$= \frac{f_0 V\left[\frac{1}{V - V_s} - \frac{1}{V + V_s}\right]}{\frac{f_0 V}{V - V_s}} \times 100$$

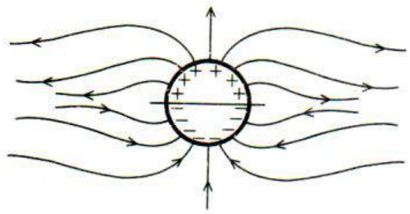
$$= \frac{2V_s}{V + V_s} \times 100 = \frac{2 \times 20}{340} \times 100 = 12\% \text{ (approx.)}$$

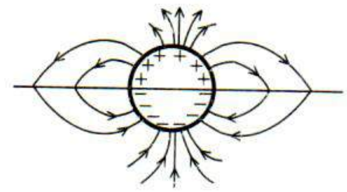
14. A long cylindrical shell carries positive surface charge σ in the upper half and negative surface charge $-\sigma$ in the lower half. The electric field lines around the cylinder will look like figure given in :

(figure are schematic and not drawn to scale)

(1) 

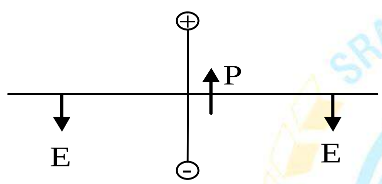
(2) 

(3) 

(4) 

Ans. (1)

Sol: The shell can be considered as a dipole as shown



So, at equatorial points the electric lines has to opposite to $\vec{P}$. This satisfied only in figure 1.

15. A uniformly charged solid sphere of radius R has potential V_0 (measured with respect to ∞) on its surface. For this sphere the equipotential surfaces with potentials $\frac{3V_0}{2}, \frac{5V_0}{4}, \frac{3V_0}{4}$ and $\frac{V_0}{4}$ have radius R_1, R_2, R_3 and R_4 respectively. Then

(1) $R_1 = 0$ and $R_2 > (R_4 - R_3)$
(2) $R_1 \neq 0$ and $(R_2 - R_1) > (R_4 - R_3)$
(3) $R_1 = 0$ and $R_2 < (R_4 - R_3)$
(4) $2R < R_4$

Ans. (3), (4)

Sol: For uniformly charged solid sphere at internal points the potential is greater than that on the surface $V_{surface} = V_0 = \frac{KQ}{R}$

$$V_{int}(r \le R) = \frac{KQ}{2R^3}\left[3R^2 - r^2\right]$$

At center (r = 0); $V = \frac{3}{2}\frac{KQ}{R} = \frac{3}{2}V_0$

$\therefore$ at $R_1 = 0$, $V_1 = \frac{3V_0}{2}$

At R_2;

$$\frac{KQ}{2R^3}\left(3R^2 - R_2^2\right) = \frac{5}{4}V_0 = \frac{5}{4}\frac{KQ}{R}$$

$$\Rightarrow \left(3R^2 - R_2^2\right) = \frac{5}{2}R^2$$

$$\Rightarrow \frac{R^2}{2} = R_2^2 \Rightarrow R_2 = \frac{R}{\sqrt{2}}$$

At R_3; $V = \frac{3}{4}V_0$ (which is less then V_0) So R_3 should be outside the sphere.

$$\frac{KQ}{R_3} = \frac{3}{4}\frac{KQ}{R} \Rightarrow R_3 = \frac{4R}{3}$$

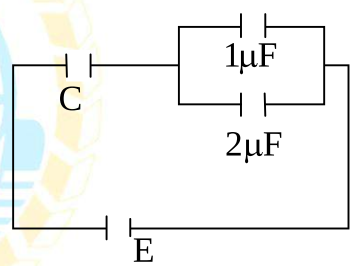
Similarly at R_4; $\frac{KQ}{R_4} = \frac{1}{4}\frac{KQ}{R} \Rightarrow R_4 = 4R$

Now, $R_4 - R_3 = 4R - \frac{4R}{3} = \frac{8R}{3}$

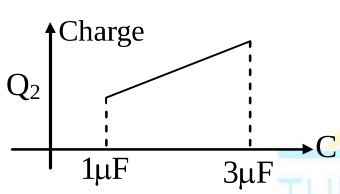
So, $R_2 < \left(R_4 - R_3\right)$

So, option (3) is correct. $2R < R_4$ hence option (4) is also correct.

16. In the given circuit, charge Q_2 on the $2\mu F$ capacitor changes as C is varied from $1\mu F$ to $3\mu F$. Q_2 as a function of 'C' is given properly by : (figures are drawn schematically and are not to scale)



(1)



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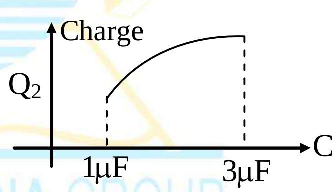

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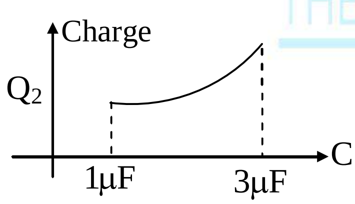

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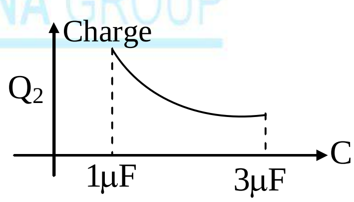
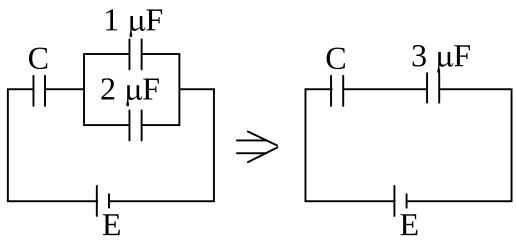


Ans. (2)

Sol:



$$C_{eq} = \frac{3C}{C+3}$$

$$Q = C_{eq}.E = \frac{3CE}{C+3}$$

$$\therefore \quad Q_2 = \frac{Q \times 2}{3} = \frac{2CE}{C+3}$$

$$\frac{dQ_2}{dC} = \left[\frac{(C+3)2E - 2CE}{(C+3)^2}\right] = \frac{6E}{(C+3)^2}$$

Since $\frac{dQ_2}{dC}$ is decreasing with increase in value of C

$\therefore$ option (2) is correct.

17. When 5V potential difference is applied across a wire of length 0.1 m, the drift speed of electrons is $2.5 \times 10^{-4} ms^{-1}$. If the electron density in the wire is $8 \times 10^{28} m^{-3}$, the resistivity of the material is closed to:

(1) $1.6 \times 10^{-8} \Omega m$ (2) $1.6 \times 10^{-7} \Omega m$ (3) $1.6 \times 10^{-6} \Omega m$ (4) $1.6 \times 10^{-5} \Omega m$

Ans. (4)

Sol: $j = \sigma E$

$\frac{I}{A} = \sigma \frac{V}{l}$ (1) where $I = neAV_d$,

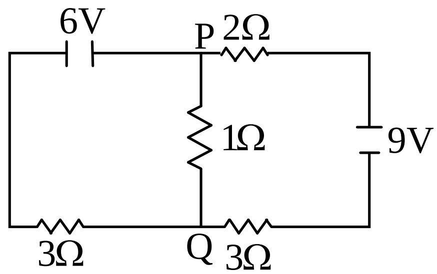
Now, $\frac{I}{A} = neV_d$

From (1), $neV_d = \sigma \frac{V}{l} \Rightarrow neV_d = \frac{1}{\rho}\frac{V}{l}$

$$\rho = \frac{V}{neV_d\, l} = \frac{5}{8 \times 10^{28} \times 1.6 \times 10^{-19} \times 2.5 \times 10^{-4} \times 0.1}$$

$\rho = 1.6 \times 10^{-5}\ \Omega m$.

18.



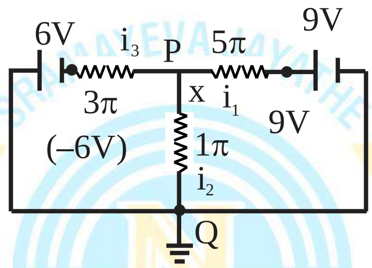
In the circuit shown, the current in the 1Ω resistor is:

(1) 1.3 A, from P to Q (2) 0 A

(3) 0.13 A, from Q to P (4) 0.13 A, from P to Q

Ans. (3)

Sol:



$i_1 = i_2 + i_3$

$\frac{9-x}{5} = \frac{x+6}{3} + \frac{x}{1}$

$27 - 3x = 5x + 30 + 15x$

$x = \frac{-3}{23}$

Current is (3/23) Amp from Q to P.

19. Two coaxial solenoids of different radii carry current I in the same direction. Let $\vec{F_1}$ be the magnetic force on the inner solenoid due to the outer one and $\vec{F_2}$ be the magnetic force on the outer solenoid due to the inner one. Then :

(1) $\vec{F_1} = \vec{F_2} = 0$

(2) $\vec{F_1}$ is radially inwards and $\vec{F_2}$ is radially outwards

(3) $\vec{F_1}$ is radially inwards and $\vec{F_2} = 0$

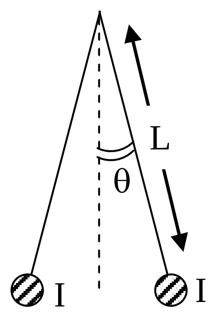
(4) $\vec{F_1}$ is radially outwards and $\vec{F_2} = 0$

Ans. (1)

Sol: Magnetic field due to the inner coil at external points will be zero.
So magnetic force (iLB) on the outer coil will be zero.
The magnetic field at any current element of inner coil due to outer coil is perpendicular to it.
As per Fleming's Left Hand rule (or $d\vec{F} = I\left(d\vec{l} \times \vec{B}\right)$ the force will be radially.

So net force on inner coil is also zero. (Every loop of inner coil is a uniform magnetic field)
So. F = 0

20. Two long current carrying thin wires, both with current I, are held by insulating threads of length L and are in equilibrium as shown in the figure, with threads making an angle 'θ' with the vertical. If wires have mass λ per unit length then the value of I is:
(g = gravitational acceleration)



(1) $\sin\theta\sqrt{\dfrac{\pi\lambda gL}{\mu_0\cos\theta}}$ (2) $2\sin\theta\sqrt{\dfrac{\pi\lambda gL}{\mu_0\cos\theta}}$

(3) $2\sqrt{\dfrac{\pi gL}{\mu_0}\tan\theta}$ (4) $\sqrt{\dfrac{\pi\lambda gL}{\mu_0}\tan\theta}$

Ans. (2)

Sol:

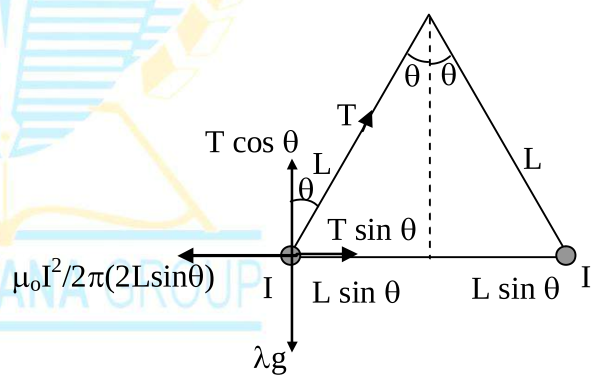
Assume length of the wire is 1 m
Since the wire is in equilibrium ,

T sin θ = $\dfrac{\mu_o I^2}{4\pi L\sin\theta}$ (1)

T cos θ = λg (2)

On dividing (1) and (2),

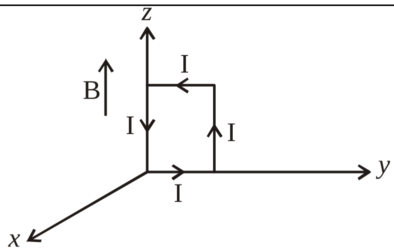
tan θ = $\dfrac{\mu_o I^2}{4\pi L\sin\theta.\lambda g}$

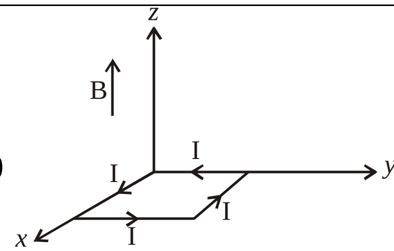


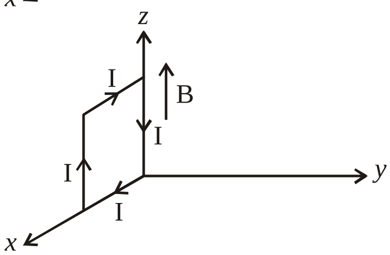
Hence, I = $\sqrt{\dfrac{4\pi L\lambda g\sin\theta\tan\theta}{\mu_o}} = \sqrt{\dfrac{4\pi L\lambda g\sin^2\theta}{\mu_o\cos\theta}}$

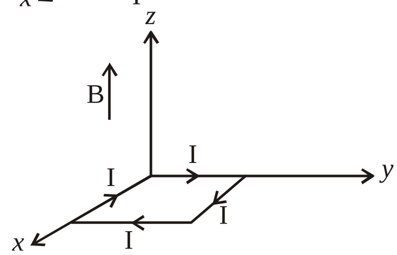
Hence, I = $2\sin\theta\sqrt{\dfrac{\pi L\lambda g}{\mu_o\cos\theta}}$

21. A rectangular loop of sides 10 cm and 5cm carrying a current I of 12 A is placed in different orientations as shown in the figures below:

(a) 

(b) 

(c) 

(d) 

If there is a uniform magnetic field of 0.3 T in the positive *z* direction, in which orientations the loop would be in (i) stable equilibrium and (ii) unstable equilibrium?

(1) (a) and (b), respectively
(2) (a) and (c), respectively
(3) (b) and (d), respectively
(4) (b) and (c), respectively

Key: (3)

Sol. $\tau = MB\sin\theta$

$U = -MB\cos\theta$

In (a) θ = 90

(b) θ = 0^0

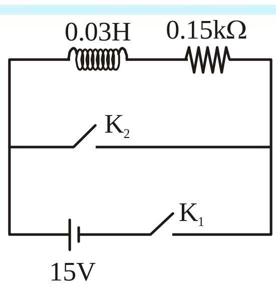
(c) θ = 90

(d) θ 180

So, (b) is stable equilibrium

& (d) is unstable

22. An inductor (L = 0.03H) and a resistor (R = 0.15 kΩ) are connected in series to a battery of 15V EMF in a circuit shown below. The key K_1 has been kept closed for a long time. Then at t = 0, K_1 is opened and key K_2 is closed simultaneously. At t = 1ms, is closed in the circuit will be $\left(e^5 \cong 150\right)$



(1) 100 mA
(2) 67 mA
(3) 6.7 mA
(4) 0.67 mA

Key: (4)

Sol. At t= ∞ when K_1 is closed

$$I_0 = \frac{15}{150} = \frac{1}{10} A.$$

When K_1 is opened & K_2 is closed.

$$I = I_0 e^{-t/\tau} \ \& \ \tau = \frac{L}{R} = \frac{0.03}{0.15\times10^3} = 2\times10^{-4}$$

$$\therefore \quad I = \frac{1}{10} e^{-\frac{10^{-3}}{2\times10^{-4}}}$$

$$= \frac{1}{10} e^{-5}$$

$$= \frac{1}{10} e^{-5} \qquad = 0.67mA$$

23. A red LED emits light at 0.1 watt uniformly around it. The amplitude of the electric field of the light at a distance of 1m from the diode is:

(1) 1.73 V/m (2) 2.45 V/m (3) 5.48 V/m (4) 7.75 V/m

Key: (2)

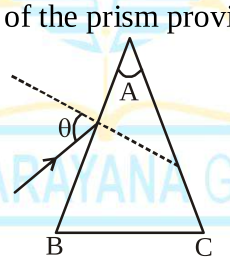
Sol. $P = \left(\frac{1}{2}\varepsilon_0 E_0^2\right) \times A \times C$

$$P = \left(\frac{1}{2}\varepsilon_0 E_0^2\right) \times A \times C$$

$$= \sqrt{\frac{2\times0.1}{8.85\times10^{-12}\times4\pi\times1^2\times3\times10^8}}$$

$$= 2.45\, V/m$$

24. Monochromatic light is incident on a glass prism of angle A. If the refractive index of the material of the prism is μ, a ray, incident at an angle θ, on the face AB would get transmitted through the face AC of the prism provided:

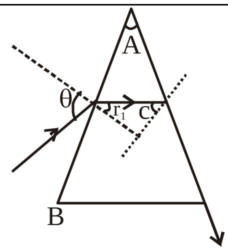


(1) $\theta > \sin^{-1}\left[\mu \sin\left(A - \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$ (2) $\theta < \sin^{-1}\left[\mu \sin\left(A - \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$

(3) $\theta > \cos^{-1}\left[\mu \sin\left(A + \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$ (4) $\theta < \cos^{-1}\left[\mu \sin\left(A + \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$

Key: (1)

Sol.



$A = r_1 + C$

$r_1 = (A - C)$

Using snell's law on first face

$\sin\theta = \mu \sin r_1$

$= \mu \sin(A - C)$

$= \mu \sin\left(A - \sin^{-1}\frac{1}{\mu}\right)$

$\Rightarrow \theta = \sin^{-1}\left[\mu \sin\left(A - \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$

For transmission through face AC

$\theta > \sin^{-1}\left[\mu \sin\left(A - \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$

25. On a hot summer night, the refractive index of air smallest near the ground and increases with height from the ground. When a light beam is directed horizontally, the Huygen's principle leads us to conclude that as it travels, the light beam:

(1) becomes narrower
(2) goes horizontally without any deflection
(3) bends downwards
(4) bends upwards

Key: (4)

Sol. As R.I increases ray bend towards normal, hence it will bend upward.

26. Assuming human pupil to have a radius of 0.25 cm and a comfortable viewing distance of 25 cm, the minimum separation between two objects that human eye can resolve at 500 nm wavelength is:

(1) 1 μm
(2) 30 μm
(3) 100 μm
(4) 300 μm

Key: (2)

Sol. $\frac{1.22\lambda}{d} \le \frac{y}{D}$

$\frac{1.22 \times 500 \times 10^{-9}}{0.50 \times 10^{-2}} \le \frac{y}{25 \times 10^{-2}}$

$y = 30.50 \times 10^{-6} m$

$\simeq 30\ \mu m$

27. As an electron makes a transition from an excited state to the ground state of a hydrogen-like atom/ion:
(1) its kinetic energy increases but potential energy and total energy decrease
(2) kinetic energy, potential energy and total energy decrease
(3) kinetic energy decreases, potential energy increases but total energy remains same
(4) kinetic energy and total energy decrease but potential energy increase

Key: (1)

Sol. $KE = \frac{kZe^2}{2r}$

$PE = \frac{-kZe^2}{r}$

$TE = \frac{-kZe^2}{2r}$

28. Match **List-I** (Fundamental Experiment) with **List-II** (its conclusion) and select the correct option from the choices given below the list:

	List-I		**List-II**
(A)	Franck-Herts Experiment.	(i)	Particle nature
(B)	Photo-electric experiment	(ii)	Discrete energy levels of atom
(C)	Davision-Germer Experiment.	(iii)	Wave nature of electron
		(iv)	Structure of atom

(1) (A) – (i) (B) – (iv) (C)-(iii)
(2) (A) – (ii) (B) – (iv) (C) – (iii)
(3) (A) – (ii) (B) – (i) (C) – (iii)
(4) (A) – (iv) (B) – (iii) (C) – (ii)

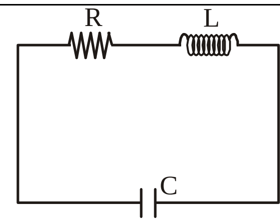
Key: (3)

29. A signal of 5kHz frequency is amplitude modulated on a carrier wave of frequency 2 MHz. The frequency of the resultant signal is/are:
(1) 2 KHz only (2) 2005 kHz, and 1995 kHz
(3) 2005 kHz, 2000 kHz and 1995 kHz (4) 2000 kHz and 1995 kHz

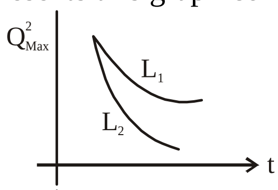
Key: (3)

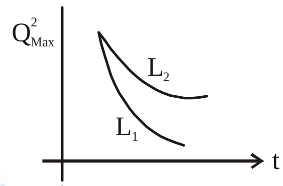
Sol. Resultant frequency of signal if given by $(f_C - f_M), (f_R), (f_C + f_M)$
Then possible frequencies are $2005\,kHz, 2000\,kGz, 1995\,kHz$

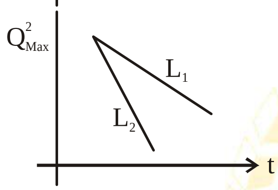
30. An LCR circuit is equivalent to a damped pendulum. In an LCR circuit the capacitor is charged to Q_0 and then connected the L and R as shown below:

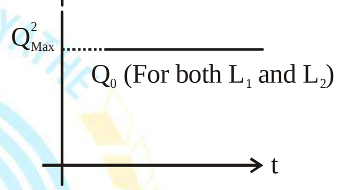


If a student plots graphs of the square of maximum charge $\left(Q^2_{Max}\right)$ on the capacitor with time (t) for two different values L_1 and $L_2 (L_1 > L_2)$ of L then which of the following represents this graph correctly? (plots are schematic and not drawn to scale)

(1)


(2)


(3)


(4)


Key. (1)

Sol: $\frac{q}{C} - IR - L\frac{dI}{dt} = 0$

$\frac{q}{C} - \left(\frac{dq}{dt}\right)R = L\left(\frac{dI}{dt}\right)$

Comparing with equation of damped oscillation

$F_{damp} = -bv = -\left(\frac{R}{L}\right)$

Hence, $b \propto \frac{R}{L}$

$\Rightarrow \quad b \propto \frac{1}{L}$

As $A = A_0 e^{-\left(\frac{b}{2m}\right)t}$

$Q = Q_0 e^{-\left(\frac{b}{2m}\right)t}$

$Q^2 = Q_0^2 e^{-\left(\frac{b}{m}\right)t}$

Topic & Level

Q.No.	**Topic**	**Level**
1.	Kinematics	Medium
2.	Experimental Physics	Easy
3.	Friction	Medium
4.	Collision	Easy
5.	Centre of Mass	Medium
6.	Rotational Motion	Difficult
7	Gravitation	Difficult
8.	Elasticity	Difficult
9.	Heat & Thermodynamics	Difficult
10.	Heat & Thermodynamics	Medium
11.	Heat & Thermodynamics	Difficult
12.	Simple Harmonic Motion	Easy
13.	Waves	Easy
14.	Electrostatics	Medium
15.	Electrostatics	Difficult
16.	Electrostatics	Difficult
17.	Current Electricity	Easy
18.	Current Electricity	Easy
19.	Magnetism	Easy
20.	Magnetism	Medium
21.	Magnetism	Easy
22.	E.M.I.	Easy
23.	Electro magnetic waves	Easy
24.	Optics	Medium
25.	Optics	Easy
26.	Optics	Difficult
27.	Modern Physics	Easy
28.	Modern Physics	Easy
29.	Modern Physics	Easy
30.	E.M.I.	Difficult

S.No.	**Major Topic**	**No. of Question**
1	Mechanics	**8**
2	Heat & Thermodynamics	**3**
3	S.H.M. & Waves	**2**
4	Electricity & Magnetism	**9**
5	E.M.Waves & Optics	**4**
6	Modern Physics	**4**
	Total	**30**

CHEMISTRY

31. The molecular formula of a commercial resin used for exchanging ions in water softening is $C_8H_7SO_3Na$ (Mol.wt.206). What would be the maximum uptake of Ca^{2+} ions by the resin when expressed in mole per gram resin?

(1) $\frac{1}{103}$ (2) $\frac{1}{206}$ (3) $\frac{2}{309}$ (4) $\frac{1}{412}$

Key. (4)

Sol. Na^+ ion of resin will be exchanged with Ca^{2+} ion

$$2C_8H_7SO_3Na + Ca^{2+} \longrightarrow (C_8H_7SO_3)_2Ca + 2Na^+$$

So maximum uptake of Ca^{2+} ion by the resin $= \frac{1}{206 \times 2} = \frac{1}{412} \text{mol g}^{-1}$ of resin.

32. Sodium metal crystallizes in a body centered cubic lattice with a unit cell edge of $4.29\,\mathring{A}$. The radius of sodium atom is approximately :

(1) $1.86\,\mathring{A}$ (2) $3.22\,\mathring{A}$ (3) $5.72\,\mathring{A}$ (4) $0.93\,\mathring{A}$

Key. (1)

Sol. For B.c.c. unit cell

$4r = \sqrt{3}a$

$\therefore\ r = \frac{\sqrt{3} \times 4.29}{4} = 1.86\,\mathring{A}$

33. Which of the following is the energy of a possible excited state of hydrogen?

(1) $+13.6\,eV$ (2) $-6.8\,eV$ (3) $-3.4\,eV$ (4) $+6.8\,eV$

Key. (3)

Sol. Energy of second shell of hydrogen $= -\frac{13.6 \times 1^2}{2^2} = -3.4\,eV/\text{atom}$

34. The intermolecular interaction that is dependent on the inverse cube of distance between the molecules is

(1) ion – ion interaction (2) ion – dipole interaction

(3) London force (4) hydrogen bond

Key. (4)

Sol. Dipole –dipole interaction energy between stationary polar molecules (as in solids) is proportional to $\frac{1}{r^3}$.

Hydrogen bond is a special case of dipole-dipole interactions.

35. The following reaction is performed at 298 K.

$$2NO(g) + O_2(g) \rightleftharpoons 2NO_2(g)$$

The standard free energy of formation of NO(g) is 86.6 kJ/mol at 298 K. What is the standard free energy of formation of NO_2(g) at 298 K? $(K_p = 1.6 \times 10^{12})$

(1) $R(298)\ln(1.6 \times 10^{12})$ (2) $86600 + R(298)\ln(1.6 \times 10^{12})$

(3) $86600 - \frac{\ln\left(1.6\times10^{12}\right)}{R\left(298\right)}$ (4) $0.5\left[2\times86600 - R\left(298\right)\ln\left(1.6\times10^{12}\right)\right]$

Key. (4)

Sol. $\Delta G_R = -RT\ln K_p$

$\Delta G_R = 2\left[\Delta_f G\left(NO_2\right) - \Delta_f G\left(NO\right)\right]$

$\Delta G_f\left(NO_2\right) = \Delta G_R + \frac{\Delta_f G\left(NO\right)}{2}$

$= 86.6\times10^3 - \frac{R\left(298\right)\ln 1.6\times10^{12}}{2}$

36. The vapour pressure of acetone at 20°C is 185 torr. When 1.2 g of a non-volatile substance was dissolved in 100 g of acetone at 20°C, its vapour pressure was 183 torr. The molar mass (g mol^{-1}) of the substance is

(1) 32 (2) 64 (3) 128 (4) 488

Key. (2)

Sol. $\frac{P^o - P^5}{P^0} = X_{solute}$

$\frac{185-183}{P^o} = \frac{\frac{1.2}{M}}{\frac{1.2}{M}+\frac{100}{58}}$

$\Rightarrow \frac{2}{185} = \frac{1.2}{M}\times\frac{58}{100}$ $\left(\text{Assume } \frac{1.2}{M} << \frac{100}{58}\right)$

$\Rightarrow M = \frac{1.2\times58\times185}{2\times100} = 64.38$

37. The standard Gibbs energy change at 300 K for the reaction $2A \rightleftharpoons B + C$ is 2494.2 J. At a given time, the composition of the reaction mixture is $\left[A\right] = \frac{1}{2}, \left[B\right] = 2$ and $\left[C\right] = \frac{1}{2}$. The reaction proceeds in the : [R = 8.314 J/K/mol, e = 2.718]

(1) forward direction because $Q > K_C$ (2) reverse direction because $Q > K_c$

(3) forward direction because $Q < K_c$ (4) reverse direction because $Q < K_c$

Key. (2)

Sol. $\Delta G^o = -RT\ln K_c$

$2494.2 = -8.314\times300\ln K_c$

$\ln K_c = -1$

$K_c = e^{-1} = \frac{1}{e} = \frac{1}{2.718}$

$Q_c = \frac{[B][C]}{[A]^2}$

$$= \frac{2\times\frac{1}{2}}{\left(\frac{1}{2}\right)^2} = 4$$

$\therefore$ Reverse direction as $Q_c > K_c$

38. Two Faraday of electricity is passed through a solution of $CuSO_4$. The mass of copper deposited at the cathode is : (at.mass of Cu = 63.5 amu)

(1) 0 g (2) 63.5 g (3) 2 g (4) 127 g

Key. (2)

Sol. 1 Faraday of electricity deposits 1 equivalent Cu
2 Faraday of electricity deposits 2 equivalents of Cu
1 mol of Cu = 63.5 g

39. Higher order (>3) reactions are rare due to

(1) low probability of simultaneous collision of all the reacting species
(2) increase in entropy and activation energy as more molecules are involved
(3) shifting of equilibrium towards reactants due to elastic collisions
(4) loss of active species on collision

Key. (1)

Sol. Higher order (>3) reactions are rare due to low probability of simultaneous collision of all the reacting species.

40. 3 g of activated charcoal was added to 50 ml. of acetic acid solution (0.06 N) in a flask. After an hour it was filtered and the strength of the filtrate was found to be 0.042 N. The amount of acetic acid adsorbed (per gram of charcoal) is

(1) 18 mg (2) 36 mg (3) 42 mg (4) 54 mg

Key. (1)

Sol. No. of millimols of acetic acid before adsorption $= 50\times 0.06 = 3$
No. of millimols of acetic acid after adsorption = $50\times 0.042 = 2.1$
No. of millimols of acetic acid adsorbed = $3 - 2.1 = 0.9$

Weight of acetic acid adsorbed $= \frac{0.9\times 60}{10^3}\,g = 54\,mg$

Amount of acetic acid adsorbed per gm of charcoal = $\frac{54}{3}$ mg = 18 mg

41. The ionic radii (in Å) of N^{3-}, O^{2-} and F^- are respectively

(1) 1.36, 1.40 and 1.71 (2) 1.36, 1.71 and 1.40
(3) 1.71, 1.40 and 1.36 (4) 1.71, 1.36 and 1.40

Key: (3)

Sol: $N^{-3} > O^{-2} > F^-$
As the charge increase on anion its size increases due to decrease in Z_{eff}.

42. In the context of the Hall – Heroult process for the extraction of Al, which of the following statements is false?

(1) CO and CO_2 are produced in this process
(2) Al_2O_3 is mixed with CaF_2 which lowers the melting point of the mixture and brings conductivity

(3) Al^{3+} is reduced at the cathode to form Al
(4) Na_3AlF_6 serves as the electrolyte

Key: (4)
Sol: CaF_2 or Na_3AlF_6 reduce melting point and increase the conductivity.

43. From the following statements regarding H_2O_2, choose the incorrect statement:
(1) It can act only as an oxidizing agent
(2) It decomposes an exposure to light
(3) It has to be stored in plastic or wax lined glass bottle in dark
(4) It has to be kept away from dust

Key: (1)
Sol: It act as both OA/RA due to intermediate OS of oxygen.

44. Which one of the following alkaline earth metal sulphates has its hydration enthalpy greater than its lattice enthalpy?
(1) $CaSO_4$ (2) $BeSO_4$ (3) $BaSO_4$ (4) $SrSO_4$

Key: (2)
Sol: $BeSO_4$ soluble in water, the hydration energy greater than its lattice energy.

45. Which among the following is the most reactive?
(1) Cl_2 (2) Br_2 (3) I_2 (4) ICl

Key: (4)
Sol: Inter Halogen compounds are more reactive due to improper overlapping than halogen.

46. Match the catalysts to the correct process:

Catalyst	Process
(A) $TiCl_3$	(i) Wacker process
(B) $PdCl_2$	(ii) Ziegler – Natta Polymerizatio n
(C) $CuCl_2$	(iii) Contact process
(D) V_2O_5	(iv) Deacon's process

(1) (A) – (iii), (B) – (ii), (C) – (iv), (D) – (i) (2) (A) – (ii), (B) – (i), (C) – (iv), (D) – (iii)
(3) (A) – (ii), (B) – (iii), (C) – (iv), (D) – (i) (4) (A) – (iii), (B) – (i), (C) – (ii), (D) – (iv)

Key: (2)
Sol: $TiCl_3 + (C_2H_5)_3Al$ Ziegler Natta Catalyst
$PdCl_2$ used in Wacker process.
$HCl + O_2 \xrightarrow{CuCl_2} H_2O + Cl_2$ Decan process
$SO_2 + O_2 \xrightarrow{V_2O_5} SO_3$ Contact process.

47. Which one has the highest boiling point?
(1) He (2) Ne (3) Kr (4) Xe

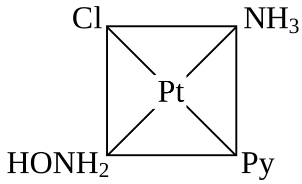
Key: (4)
Sol: Boiling point of inert gas increases down the group due to Vander Waal force increases with molar mass.

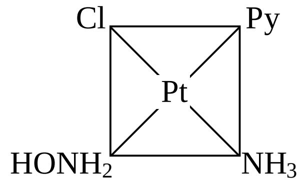
48. The number of geometric isomers that can exist for square planar $[Pt(Cl)(py)(NH_3)(NH_2OH)]^+$ is (py – pyridine)

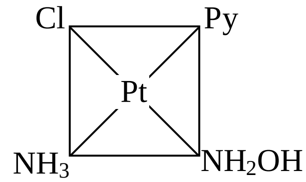
(1) 1 (2) 3 (3) 4 (4) 6

Key: (2)

Sol: planar $[Pt(Cl)(py)(NH_3)(NH_2OH)]$







49. The color of $KMnO_4$ is due to

(1) M $\rightarrow$ L charge transfer transition

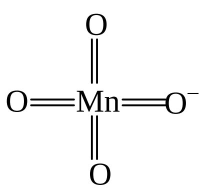
(2) d – d transition

(3) L $\rightarrow$ M charge transfer transition

(4) σ - σ^* transition

Key: (3)

Sol: MnO_4^-



Color is due to charge transfer from oxide ion to empty ‘d’ orbital of Mn.

50. Assertion : Nitrogen and Oxygen are the main components in the atmosphere but these do not react to form oxides of nitrogen.

Reason: The reaction between nitrogen and oxygen requires high temperature.

(1) Both assertion and reason are correct, and the reason is the correct explanation for the assertion

(2) Both assertion and reason are correct, but the reason is not the correct explanation for the assertion.

(3) The assertion is incorrect, but the reason is correct.

(4) Both the assertion and reason are incorrect

Key: (1)

Sol: The bond enthalpy of N_2 is much higher hence the process is endothermic.

$N_2 + O_2 \rightarrow 2NO$ - Heat

51. In Carius method of estimation of halogen, 250 mg of an organic compound gave 141 mg of AgBr. The percentage of bromine in the compound is: *(at. Mass Ag = 108, Br = 80)*

(1) 24 (2) 36 (3) 48 (4) 60

Key (1)

Sol. Mass of AgBr precipitated 0.141 g

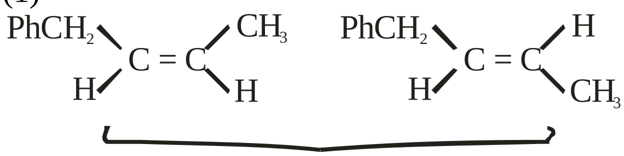
% of Br = $\frac{80 \times 0.141}{188 \times 0.250} \times 100 = 24$

52. Which of the following compounds will exhibit geometrical isomerism?

(1) 1 – Phenyl – 2 – butene (2) 3 – Phenyl – 1 – butene
(3) 2 – Phenyl – 1 – butene (4) 1, 1, - Diphenyl – 1 – propene

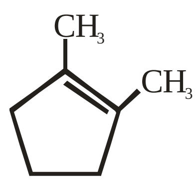
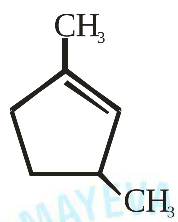
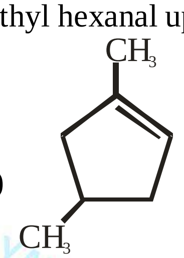
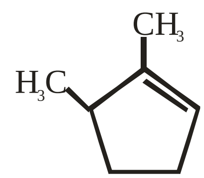
Key (1)

Sol.



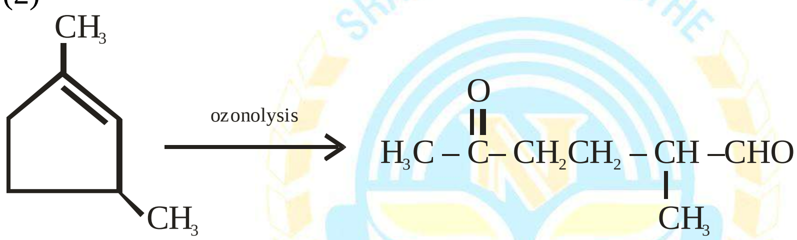
Geometrical Isomers

53. Which compound would give 5 – keto – 2 – methyl hexanal upon ozonolysis?

(1) 
(2) 
(3) 
(4) 

Key (2)

Sol.



5 - keto - 2 methyl - hexanal

54. The synthesis of alkyl fluorides is best accomplished by :
(1) Free radical fluorination (2) Sandmeyer's reaction
(3) Finkelstein reaction (4) Swarts reaction

Key (4)

Sol. $H_3C-Br+AgF\xrightarrow[\text{reaction}]{\text{Swart}}CH_3-F+AgBr$

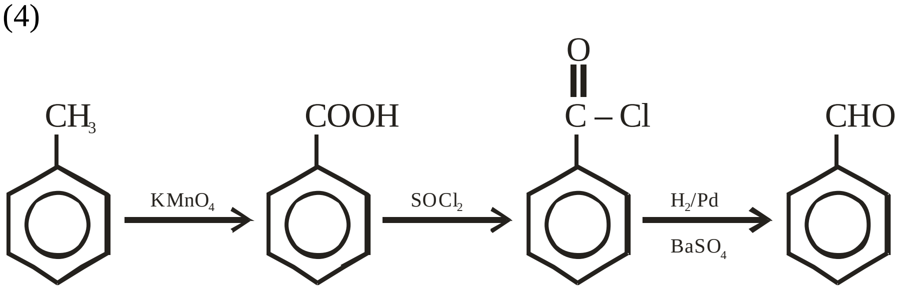
55. In the following sequence of reaction :

$\text{Toluene}\xrightarrow{KMnO_4}A\xrightarrow{SOCl_2}B\xrightarrow[BaSO_4]{H_2/Pd}C$

The product C is
(1) C_6H_5COOH (2) $C_6H_5CH_3$ (3) $C_6H_5CH_2OH$ (4) C_6H_5CHO

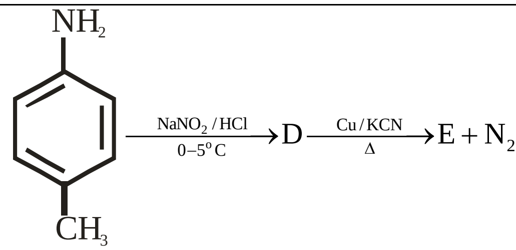
Key (4)

Sol.

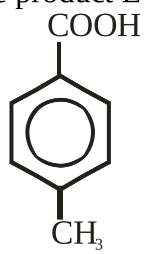


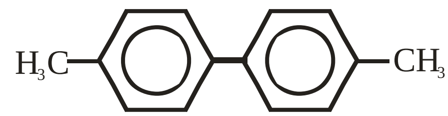
(A) (B)

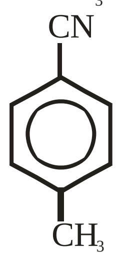
56. In the reaction



The product E is

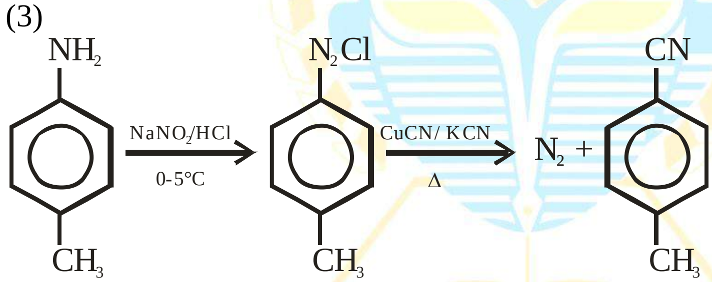
(1) 

(2) 

(3) 

(4) 

Key (3)

Sol. 

(D) (E)

57. Which polymer is used in the manufacture of paints and lacquers?

(1) Bakelite (2) Glyptal

(3) Polypropene (4) Poly vinyl chloride

Key (2)

Sol. Glyptal is used as paints and lacquers

58. Which of the vitamins given below is water soluble

(1) Vitamin C (2) Vitamin D (3) Vitamin E (4) Vitamin K

Key (1)

Sol. Vitamin A, D, E and K are fat Soluble. Vitamin C is water soluble.

59. Which of the following compounds is not an antacid?

(1) Aluminium hydroxide (2) Cimetidine

(3) Phenelzine (4) Ranitidine

Key (3)

Sol. Phenelzine is antidepressant, so not an antacid. All others are antacid

60. Which of the following compounds is not colored yellow?

(1) $Zn_2[Fe(CN)_6]$ (2) $K_3[Co(NO_2)_6]$

(3) $(NH_4)_3[As(Mo_3O_{10})_4]$ (4) $BaCrO_4$

Key (1)

Sol. $Zn_2[Fe(CN)_6]$ is bluish white.

Q. NO.	TOPIC	LEVEL
31	**Physical Chemistry**	**Simple**
32	**Physical Chemistry**	**Medium**
33	**Physical Chemistry**	**Simple**
34	**Physical Chemistry**	**Simple**
35	**Physical Chemistry**	**Medium**
36	**Physical Chemistry**	**Medium**
37	**Physical Chemistry**	**Difficult**
38	**Physical Chemistry**	**Simple**
39	**Physical Chemistry**	**Simple**
40	**Physical Chemistry**	**Medium**
41	**Inorganic Chemistry – Periodic table**	**Simple**
42	**Inorganic Chemistry – Metallurgy of Al**	**Medium**
43	**Inorganic Chemistry – Hydrogen and its compounds**	**Simple**
44	**Inorganic Chemistry (Alkaline earth Metals)**	**Medium**
45	**Inorganic Chemistry – Halogens**	**Simple**
46	**Inorganic Chemistry – Transition elements**	**Medium**
47	**Inorganic Chemistry – 18th group**	**Simple**
48	**Inorganic Chemistry – Coordination compounds**	**Medium**
49	**Inorganic Chemistry – d-block elements**	**Medium**
50	**Inorganic Chemistry – p block elements**	**Simple**
51	**Organic Chemistry**	**Medium**
52	**Organic Chemistry**	**Simple**
53	**Organic Chemistry**	**Simple**
54	**Organic Chemistry**	**Simple**
55	**Organic Chemistry**	**Simple**
56	**Organic Chemistry**	**Medium**
57	**Organic Chemistry**	**Simple**
58	**Organic Chemistry**	**Simple**
59	**Organic Chemistry**	**Medium**
60	**Organic Chemistry**	**Medium**

Organic Chemistry – 9 questions

Physical chemistry – 10 questions

Inorganic chemistry – 11 questions

MATHEMATICS

61. Let A and B be two sets containing four and two elements respectively. Then the number of subsets of the set $A\times B$, each having at least three elements is :

(1) 219 (2) 256 (3) 275 (4) 510

Key. (1)

Sol. No. of elements in $A\times B$ will be $4\times 2=8$

Now no. of subsets of $A\times B$ having at least 3 elements is

$^8C_3+{}^8C_4+{}^8C_5+....+{}^8C_8=2^8-(1+8+28)$

$=256-37=219$

62. A complex number z is said to be unimodular if $|z|=1$. Suppose z_1 and z_2 are complex numbers such that $\frac{z_1-2z_2}{2-z_1\bar{z}_2}$ is unimodular and z_2 is not unimodular. Then the point of z_1 lies on a

(1) straight line parallel to x-axis (2) straight line parallel to y-axis

(3) circle of radius 2 (4) circle of radius $\sqrt{2}$

Key. (3)

Sol. $\left|\frac{z_1-2z_2}{2-z_1\bar{z}_2}\right|=1$

$\Rightarrow |z_1-2z_2|^2=|2-z_1\bar{z}_2|^2$

$\Rightarrow (z_1-2z_2)(\bar{z}_1-2\bar{z}_2)=(2-z_1\bar{z}_2)(2-\bar{z}_1z_2)$

$\Rightarrow z_1\bar{z}_1-2z_1\bar{z}_2-2\bar{z}_1z_2+4z_2\bar{z}_2 = 4-2\bar{z}_1z_2-2z_1\bar{z}_2+z_1\bar{z}_1z_2\bar{z}_2$

$\Rightarrow |z_1|^2+4|z_2|^2=4+|z_1|^2|z_2|^2$

$\Rightarrow |z_1|^2|z_2|^2-|z_1|^2-4|z_2|^2+4=0$

$\Rightarrow \left(|z_1|^2-4\right)\left(|z_2|^2-1\right)=0$

$\Rightarrow |z_1|=2$ or $|z_2|=1$

But $|z_2|\neq 1\Rightarrow |z_1|=2$

$\Rightarrow$ z_1 lie on a circle of radius 2

63. Let α and β be the roots of equation $x^2-6x-2=0$. If $a_n=\alpha^n-\beta^n$, for $n\ge 1$, then the value of $\frac{a_{10}-2a_8}{2a_9}$ is equal to

(1) 6 (2) –6 (3) 3 (4) –3

Key. (3)

Sol. $a_n=\alpha^n-\beta^n=\alpha\alpha^{n-1}-\alpha\beta^{n-1}+\alpha\beta^{n-1}-\beta\beta^{n-1}+\beta\alpha^{n-1}-\beta\alpha^{n-1}$

$=\alpha\left(\alpha^{n-1}-\beta^{n-1}\right)+\beta\left(\alpha^{n-1}-\beta^{n-1}\right)-\alpha\beta\left(\alpha^{n-2}-\beta^{n-2}\right)$

$=(\alpha+\beta)\left(\alpha^{n-1}-\beta^{n-1}\right)-\alpha\beta\left(\alpha^{n-2}-\beta^{n-2}\right)$

$a_n=6a_{n-1}+2a_{n-2}\Rightarrow\frac{a_n-2a_{n-2}}{2a_{n-1}}=3$

Put n = 10

$\Rightarrow\frac{a_{10}-2a_8}{2a_9}=3$

64. If $A=\begin{bmatrix}1&2&2\\2&1&-2\\a&2&b\end{bmatrix}$ is a matrix satisfying the equation $AA^T=9I$, where I is 3×3 identity matrix, then the ordered pair (a,b) is equal to

(1) (2, –1) (2) (–2, 1) (3) (2, 1) (4) (–2, –1)

Key. (4)

Sol. $A=\begin{bmatrix}1&2&2\\2&1&-2\\a&2&b\end{bmatrix}$

$\Rightarrow A^T=\begin{bmatrix}1&2&a\\2&1&2\\2&-2&b\end{bmatrix}$

$A.A^T=\begin{bmatrix}1&2&2\\2&1&-2\\a&2&b\end{bmatrix}\begin{bmatrix}1&2&a\\2&1&2\\2&-2&b\end{bmatrix}$

$=\begin{bmatrix}9&0&a+4+2b\\0&9&2a+2-2b\\a+4+2b&2a+2-2b&a^2+4+b^2\end{bmatrix}=9I$

$\Rightarrow a+4+2b=0$ … (1) and $2a+2-2b=0$ … (2)

Solving (1) and (2) $a=-2, b=-1$

65. The set of all values of λ for which the system of linear equations

$2x_1-2x_2+x_3=\lambda x_1$

$2x_1-3x_2+2x_3=\lambda x_2$

$-x_1 + 2x_2 = \lambda x_3$

has a non-trivial solution,

(1) is an empty set

(2) is a singleton

(3) contains two elements

(4) contains more than two elements

Key. (3)

Sol. Rewrite the equation

$(2-\lambda)x_1 - 2x_2 + x_3 = 0$

$2x_1 - (\lambda+3)x_2 + 2x_3 = 0$

$-x_1 + 2x_2 - \lambda x_3 = 0$

For non trivial solution

$$\begin{vmatrix} 2-\lambda & -2 & 1 \\ 2 & -(\lambda+3) & 2 \\ -1 & 2 & -\lambda \end{vmatrix} = 0$$

$\Rightarrow (\lambda-1)(\lambda^2+2\lambda-3) = 0$

$\Rightarrow (\lambda-1)(\lambda+3)(\lambda-1) = 0 \Rightarrow \lambda = 1, -3$

66. The number of integers greater than 6,000 that can be formed, using the digits 3, 5, 6 7 and 8, without repetition, is :

(1) 216 (2) 192 (3) 120 (4) 72

Key. (2)

Sol. Case (i) number of '4' digit numbers will be $3\times4\times3\times2 = 72$

Case (ii) number of '5' digit numbers will be $5! = 120$

So total number $= 120 + 72 = 192$

67. The sum of coefficients of integral powers of x in the binomial expansion of $\left(1-2\sqrt{x}\right)^{50}$ is

(1) $\frac{1}{2}\left(3^{50}+1\right)$ (2) $\frac{1}{2}\left(3^{50}\right)$ (3) $\frac{1}{2}\left(3^{50}-1\right)$ (4) $\frac{1}{2}\left(2^{50}+1\right)$

Key. (1)

Sol. Let $\sqrt{x} = y$

$(1-2y)^{50} = a_0 - a_1 y + a_2 y^2 - a_3 y^3 + a_4 y^4 + ...$

Put y = 1

$1 = a_0 - a_1 + a_2 - a_3 + a_4 + ...$ … (1)

Put y = –1

$3^{50} = a_0 + a_1 + a_2 + a_3 + a_4 + ...$ … (2)

Adding (1) and (2)

$$\frac{3^{50}+1}{2}=a_0+a_2+a_4+....$$

68. If m is the A.M. of two distinct real numbers ℓ and n (ℓ, n > 1) and G_1,G_2 and G_3 are three geometric means between ℓ and n, then $G_1^4+2G_2^4+G_3^4$ equals.

(1) $4\ell^2mm$ (2) $4\ell m^2n$ (3) $4\ell mn^2$ (4) $4\ell^2m^2n^2$

Key. (2)

Sol. $\frac{\ell+n}{2}=m\Rightarrow(\ell+n)=2m$

ℓ, G_1, G_2, G_3, n are in G.P

Let common difference be r

$$n=\ell.r^4\Rightarrow r=\left(\frac{n}{\ell}\right)^{1/4}$$

$$G_1=\ell.\left(\frac{n}{\ell}\right)^{1/4}=\ell^{3/4}.n^{1/4}$$

$$G_2=\ell^{1/2}n^{1/2}$$

$$G_3=\ell^{1/4}.n^{3/4}$$

Then $G_1^4+2G_2^4+G_3^4=$

$$\ell^3n+2\ell^2n^2+\ell n^3$$

$$\ell n\left(\ell^2+2\ell n+n^2\right)$$

$$=\ell n(\ell+n)^2=4m^2\ell n$$

69. The sum of first 9 terms of the series $\frac{1^3}{1}+\frac{1^3+2^3}{1+3}+\frac{1^3+2^3+3^3}{1+3+5}+....$ is

(1) 71 (2) 96 (3) 142 (4) 192

Key. (2)

Sol. $$T_n=\frac{\sum n^3}{n^2}=\frac{n^2(n+1)^2}{4.n^2}$$

$$=\frac{(n+1)^2}{4}$$

$$T_n=\frac{n^2+1+2n}{4}$$

$$=\frac{1}{4}\left[n^2+2n+1\right]$$

$$S_n=\sum T_n=\frac{1}{4}\left[\sum n^2+2\sum n+n\right]$$

$$=\frac{1}{4}\left[\frac{n(n+1)(2n+1)}{6}+2\frac{n(n+1)}{2}+n\right]$$

$$S_9 = \frac{1}{4}\left[\frac{9\times10\times19}{6} + 2\frac{9\times10}{2} + 9\right]$$

$$= \frac{1}{4}\left[15\times19 + 9\times10 + 9\right] = 96$$

70. $\lim_{x\to0}\frac{(1-\cos 2x)(3+\cos x)}{x\tan 4x}$ is equal to

(1) 4 (2) 3 (3) 2 (4) $\frac{1}{2}$

Key. (3)

Sol. $\lim_{x\to0}\frac{(1-\cos 2x)(3+\cos x)}{x\tan 4x}$

$$= \underset{x\to0}{\text{Lt}}\frac{2\sin^2 x(3+\cos x)}{x^2\left(\frac{\tan 4x}{4x}\right).4}$$

$$= \frac{2}{4}\times4 = 2$$

71. If the function : $g(x) = \begin{cases} k\sqrt{x+1}, & 0 \le x \le 3 \\ mx+2, & 3 < x \le 5 \end{cases}$, is differentiable, then the value of k + m is

(1) 2 (2) $\frac{16}{5}$ (3) $\frac{10}{3}$ (4) 4

Key: (1)

Sol: $\because$ g(x) is continuous at x = 3

$\therefore$ $k \times \sqrt{3+1} = 3m + 2$

2k = 3m + 2 (1)

for g(x) to be differentiable,

$g'(3^-) = g'(3^+)$

$\Rightarrow \frac{k}{4} = m \Rightarrow k = 4m$ (2)

From (1) and (2), 8m = 3m + 2 and k = 4 × (2/5) = 8/5

$\Rightarrow$ m = 2/5 k + m = 10/5 = 2

Option (1) is correct.

72. The normal to the curve, $x^2 + 2xy - 3y^2 = 0$, at (1, 1) :

(1) does not meet the curve again

(2) meets the curve again in the second quadrant

(3) meets the curve again in the third quadrant

(4) meets the curve again in the fourth quadrant

Key: (4)

Sol: $2x + 2y + 2x\frac{dy}{dx} - 6y\frac{dy}{dx} = 0$

$\Rightarrow \left(\frac{dy}{dx}\right)_{(1,1)} = 1$

Hence, equation of normal, $y - 1 = -1(x - 1)$

$y - 1 = -x + 1$

$y + x = 2$ (1)

$x^2 + 2x(2 - x) - 3(2 - x)^2 = 0$

$x^2 + 4x - 2x^2 - 3(4 + x^2 - 4x) = 0$

$-4x^2 + 16x - 12 = 0$

$\Rightarrow$ $x = 1, 3$ and $y = 1, -1$

$\Rightarrow$ meets the curved again in fourth quadrant.

$\therefore$ option (4) is correct.

73. Let f(x) be a polynomial of degree four having extreme values at x = 1 and x = 2. If $\lim_{x\to 0}\left[1+\frac{f(x)}{x^2}\right] = 3$, then f(2) is equal to

(1) –8 (2) –4 (3) 0 (4) 4

Key: (3)

Sol: Let $f(x) = ax^4 + bx^3 + cx^2$

$\because$ $f'(1) = 0 \Rightarrow 4a + 3b + 2c = 0$ (1)

$f'(2) = 0 \Rightarrow 32a + 12b + 4c = 0$ (2)

$\because \lim_{x\to 0}\left[1+\frac{f(x)}{x^2}\right] = 3$

$\Rightarrow c = 2$ (3)

By using (1), (2) and (3),

$a = 1/2$, $b = -2$ and $c = 2$.

Hence, $f(x) = \frac{x^4}{2} - 2x^3 + 2x^2$

$\therefore$ $f(2) = 0$

74. The integral $\int \frac{dx}{x^2\left(x^4+1\right)^{3/4}}$ equals

(1) $\left(\frac{x^4+1}{x^4}\right)^{1/4} + C$ (2) $\left(x^4+1\right)^{1/4} + C$

(3) $-\left(x^4+1\right)^{1/4} + C$ (4) $-\left(\frac{x^4+1}{x^4}\right)^{1/4} + C$

Key: (4)

Sol: $\int \frac{dx}{x^2\left(x^2+1\right)^{3/4}} = \int \frac{dx}{x^5\left(1+\frac{1}{x^4}\right)^{3/4}}$

Put, $1 + \frac{1}{x^4} = t$, $-4x^{-5}dx = dt$,

$I = -\left(1+\frac{1}{x^4}\right)^{1/4} = \frac{-\left(x^4+1\right)^{1/4}}{x} + C$

$\therefore$ (4) is correct.

75. The integral $\int_2^4 \frac{\log x^2}{\log x^2 + \log\left(36-12x+x^2\right)}dx$ is equal to

(1) 2 (2) 4 (3) 1 (4) 6

Key: (3)

Sol: $I = \int_2^4 \frac{\log\left(x^2\right)}{\log x^2 + \log\left(36-12x+x^2\right)}$ (1)

$I = \int_2^4 \frac{\log\left((6-x)^2\right)}{\log\left((6-x)^2\right) + \log x^2}$ (2)

Adding (1) and (2), $2I = \int_2^4 dx$

$2I = 2 \Rightarrow I = 1$

$\therefore$ Option (3) is correct

76. The area (in sq. units) of the region described by $\{(x, y): y^2 \le 2x \text{ and } y \ge 4x - 1\}$ is

(1) $\frac{7}{32}$ (2) $\frac{5}{64}$ (3) $\frac{15}{64}$ (4) $\frac{9}{32}$

Key: (4)

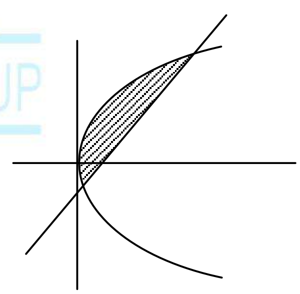
Sol: $y^2 = 2x,\ y = 4x - 1$

$2y^2 = y + 1$

$2y^2 - y - 1 = 0$

$y = 1,\ y = -1/2$

$x = ½,\ x = 1/8$



Area = $\int_{-1/2}^{1}\left(\frac{y+1}{4} - \frac{y^2}{2}\right)dy = \left[\frac{y^2}{8} + \frac{y}{4} - \frac{y^3}{6}\right]_{-1/2}^{1} =$

$\frac{9}{32}$

77. Let y(x) be the solution of the differential equation $\left(x\log x\right)\frac{dy}{dx} + y = 2x\log x, \left(x \ge 1\right)$.

then y(e) is equal to

(1) e (2) 0 (3) 2 (4) 2e

Key: (3)

Sol: $\frac{dy}{dx}+\frac{1}{x\log x}y=2$

I. F. = $e^{\int \frac{1}{x\log x}dx}=e^{\log(\log x)}=\log x$

Solving, $y\log x=\int 2.\log x.dx=2\left[(\log x)x-x\right]+C$

y. log x = 2(log x – 1)x + C, since C = 2 $\Rightarrow$ y (e) = 2

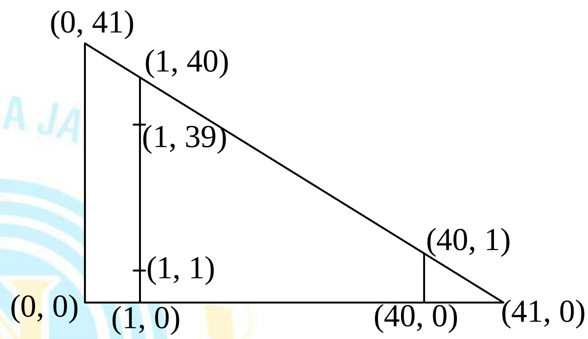
78. The number of points, having both co-ordinates as integers, that lie in the interior of the triangle with vertices (0, 0), (0, 41) and (41, 0) is

(1) 901 (2) 861 (3) 820 (4) 780

Key: (4)

Number of points = 39 + 38 + 37 + ….+ 1

$=\frac{39\times 40}{2}=780$



79. Locus of the image of the point (2, 3) in the line (2x – 3y + 4) + k(x – 2y + 3) = 0, $k\in R$, is a

(1) straight line parallel to x-axis (2) straight line parallel to y-axis

(3) circle of radius $\sqrt{2}$ (4) circle of radius $\sqrt{3}$

Key: (3)

Sol: The point of concurrency of the given family of lines is (1, 2)

So, therefore the locus of the image of the point (2, 3) in the line (2x – 3y + 4) + k(x – 2y + 3) = 0 will be a circle; whose centre is (1, 2) and radius is distance between (1, 2) and (2, 3).

Hence the equation of the required locus will be $(x-1)^2+(y-2)^2=2$.

80. The number of common tangents to the circles $x^2+y^2-4x-6y-12=0$ and $x^2+y^2+6x+18y+26=0$, is

(1) 1 (2) 2 (3) 3 (4) 4

Key: (3)

Sol: Given circles are

$x^2+y^2-4x-6y-12=0$ (1)

and $x^2+y^2+6x+18y+26=0$ (2)

$C_1=(2,3)$; $r_1=\sqrt{4+9+12}=5$

$C_2=(-3,-9)$; $r_2=\sqrt{9+81-26}=\sqrt{64}=8$

$C_1C_2=\sqrt{25+144}=13$

$C_1C_2=r_1+r_2$

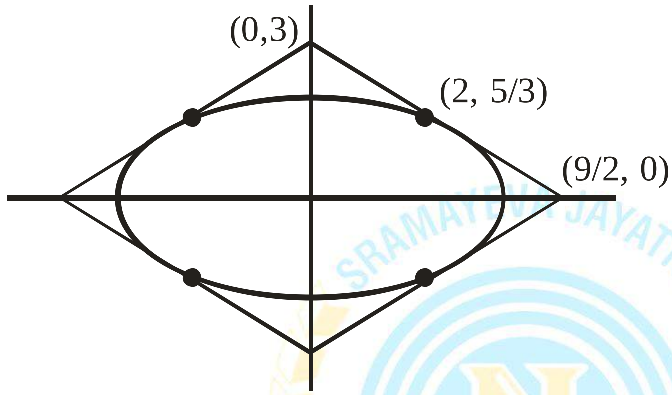
Circles touch externally each other, hence number of common tangent are three.

81. The area (in sq. units) of the quadrilateral formed by the tangents at the end points of the latera recta to the ellipse $\frac{x^2}{9}+\frac{y^2}{5}=1$, is:

(1) $\frac{27}{4}$ (2)18 (3) $\frac{27}{2}$ (4)27

Key (4)

Sol. Eq. of tangent at (2, 5/3) is $\frac{2x}{9}+\frac{y}{3}=1$



Area $=4\times\frac{1}{2}\times\frac{9}{2}\times3=27\,\text{sq.units}$

82. Let O be the vertex and Q be any point on the parabola, $x^2=8y$. If the point P divides the line segment OQ internally in the ratio 1:3, then the locus of P is:

(1) $x^2=y$ (2) $y^2=x$ (3) $y^2=2x$ (4) $x^2=2y$

Key (4)

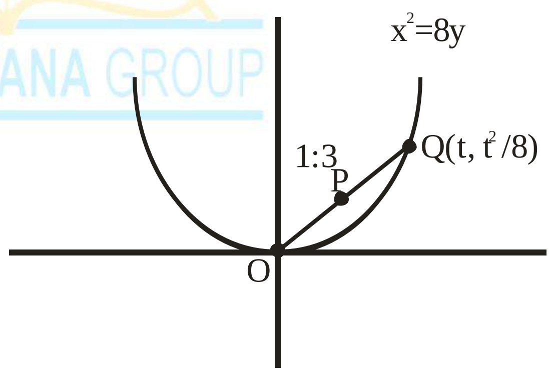
Sol. Let $P\equiv(h,k)$

$h=\frac{1.t+3\times0}{4}$ $t = 4h$ …(1)

$k=\frac{1.\frac{t^2}{8}+3\times0}{4}$ $t^2 = 32k$ …(2)

Eliminating t, we get

$x^2=2y$



83. The distance of the point (1,0,2) from the point of intersection of the line $\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{12}$ and the plane x-y+z=16, is:

(1) $2\sqrt{14}$ (2)8 (3) $3\sqrt{21}$ (4) 13

Key (4)

Sol. Let point of intersection $\equiv (3t+2, 4t-1, 12t+2)$

$\Rightarrow (3t+2)-(4t-1)+(12t+2)=16$

t=1

Point of intersection (5,3,14)

$\therefore$ distance $= \sqrt{(5-1)^2+(3-0)^2+(14-2)^2}$

=13

84. The equation of the plane containing the line 2x-5y+z=3; x+y+4z=5, and parallel to the plane, x+3y+6z=1, is :

(1) $2x+6y+12z=13$ (2) $x+3y+6z=-7$

(3) $x+3y+6z=7$ (4) $2x+6y+12z=-13$

Key (3)

Sol. Required plane is $(2x-5y+z-3)+\lambda(x+y+4z-5)=0$

Or

$x(2+\lambda)+y(\lambda-5)+z(4\lambda+1)-(3+5\lambda)=0$……..(i)

$x+3y+6z-1=0$ ………..(ii)

As (i)and (ii) are parallel

$\Rightarrow \frac{2+\lambda}{1}=\frac{\lambda-5}{3}=\frac{4\lambda+1}{6}$

$\Rightarrow \lambda=\frac{-11}{2}$

$\therefore$ Required plane is

x+3y+6z=7

85. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three non-zero vectors such that no two of them are collinear and $(\vec{a}\times\vec{b})\times\vec{c}=\frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$. If θ is the angle between vectors $\vec{b}$ and $\vec{c}$, then a value of $\sin\theta$ is:

(1) $\frac{2\sqrt{2}}{3}$ (2) $\frac{-\sqrt{2}}{3}$ (3) $\frac{2}{3}$ (4) $\frac{-2\sqrt{3}}{3}$

Key (1)

Sol. $(\vec{c}.\vec{b})\vec{a}-(\vec{c}.\vec{a})\vec{b}=-\frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$

$\Rightarrow \vec{c}.\vec{b}=-\frac{1}{3}|\vec{b}||\vec{c}|$ $(\text{and } \vec{c}.\vec{b}=0)$

$\therefore$ $\cos\theta=-\frac{1}{3}$

$\sin\theta=\frac{2\sqrt{2}}{3}$

86. If 12 identical balls are to be placed in 3 identical boxes, then the probability that one of the boxes contains exactly 3 balls is :

(1) $\frac{55}{3}\left(\frac{2}{3}\right)^{11}$ (2) $55\left(\frac{2}{3}\right)^{10}$ (3) $220\left(\frac{1}{3}\right)^{12}$ (4) $22\left(\frac{1}{3}\right)^{11}$

Key (1)

Sol. Required probability

$$=\frac{12C_3\times 2^9}{3^{12}}$$

$$=\frac{12\times 11\times 10\times 2^9}{6\times 3^{12}}=\frac{55}{3}\left(\frac{2}{3}\right)^{11}$$

87. The mean of the data set comprising of 16 observations is 16.if one of the observation valued 16 is deleted and three new observations valued 3, 4 and 5 are added to the data, then the mean of the resultant data, is:

(1) 16.8 (2)16.0 (3)15.8 (4)14.0

Key (4)

Sol. $\frac{\sum_{i=1}^{16} x_i}{16}=16$

$\therefore \quad \sum_{i=1}^{16} x_i = 256$

$\sum_{i=1}^{16} x_i -16+3+4+5=252 \quad =\frac{252}{18}=14$

88. If the angles of elevation of the top of a tower from three collinear points A,B and C, on a line leading to the foot of the tower, are 30^0, 45^0 and 60^0 respectively, then the ratio is, AB:BC, is :

(1) $\sqrt{3}:1$ (2) $\sqrt{3}:\sqrt{2}$ (3) $1:\sqrt{3}$ (4) $2:3$

Key (1)

Sol. $\tan 60^o=\frac{x}{y}\Rightarrow x=y\sqrt{3}$ ...(1)

$\tan 45^o=\frac{x}{BC+y}\Rightarrow x=BC+y$ ...(2)

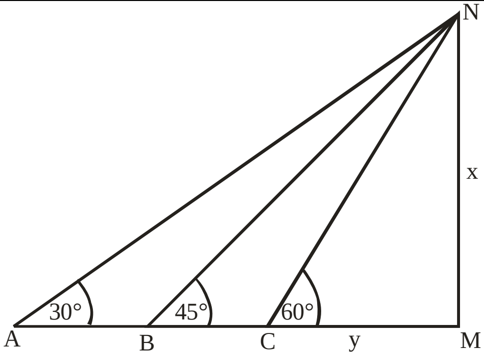
$\tan 30^o=\frac{x}{y+AC}\Rightarrow x\sqrt{3}=AC+y$

$\sqrt{3}x=y+AB+BC$ ....(3)

$y\sqrt{3}=BC+y$

$BC=y\left(\sqrt{3}-1\right)$ ...(4)

$\sqrt{3}\left(y\sqrt{3}\right)=y+AB+BC$



$AB + BC = 2y \qquad \ldots(5)$

$AB = 2y - \left(y\sqrt{3} - y\right) = 2y - \sqrt{3}y + y$

$\frac{AB}{BC} = \frac{2-\sqrt{3}+1}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = 2\sqrt{3} + 2 - 3 - \sqrt{3} + \sqrt{3} + 1 = \sqrt{3}$

89. Let $\tan^{-1} y = \tan^{-1} x + \tan^{-1}\left(\frac{2x}{1-x^2}\right)$, where $|x| < \frac{1}{\sqrt{3}}$. Then a value of y is :

(1) $\frac{3x-x^3}{1-3x^2}$ (2) $\frac{3x+x^3}{1-3x^2}$ (3) $\frac{3x-x^3}{1+3x^2}$ (4) $\frac{3x+x^3}{1+3x^2}$

Key (1)

Sol. Let $x = \tan\theta$, where $\theta \in (-\pi/6, \pi/6)$

$\tan^{-1} y = \theta + 2\theta = 3\theta$

$y = \tan 3\theta = \frac{3x-x^3}{1-3x^2}$, where $-\frac{\pi}{2} < 3\theta < \frac{\pi}{2}$

90. The negation of $\sim s \vee (\sim r \wedge s)$ is equivalent to

(1) $s \wedge \sim r$ (2) $s \wedge (r \wedge \sim s)$ (3) $s \vee (r \vee \sim s)$ (4) $s \wedge r$

Key (4)

Sol.

r	s	~s	~r	$\sim r \wedge s$	$\sim s \vee (\sim r \wedge s)$	$\sim[\sim s \vee (\sim r \wedge s)]$	$s \wedge r$
T	T	F	F	F	F	T	T
T	F	T	F	F	T	F	`F
F	T	F	T	T	T	F	F
F	F	T	T	F	T	F	F