

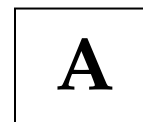
**NARAYANA IIT / NEET ACADEMY
INDIA**

This booklet contains 48 printed pages

PAPER – 1 : PHYSICS, CHEMISTRY & MATHEMATICS

Do not open this Test Booklet until you are asked to do so.

Read carefully the instructions on the Back Cover of this Test Booklet.

***Important Instructions :***

1. Immediately fill in the particulars on this page of the Test Booklet with Blue/Black Ball Point Pen. Use of pencil is strictly prohibited.
2. The Answer Sheet is kept inside this Test Booklet. When you are directed to open the Test Booklet, take out the Answer Sheet and fill in the particulars carefully.
3. The test is of 3 hours duration.
4. The Test Booklet consists of 90 questions. The maximum marks are 360.
5. There are three parts in the question paper A, B, C consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage. Each question is allotted 4 (four) marks for each correct response.
6. Candidates will be awarded marks as stated above in instructions No.5 for correct response of each question, $\frac{1}{4}$ (one fourth) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.
7. There is only one correct response for each question. Filling up more than one response in each question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 6 above.
8. For writing particulars/markings responses on Side-1 and Side-2 of the Answer Sheet use only *Black Ball Point Pen* provided in the examination hall.
9. No candidate is allowed to carry any textual material, printed or written, bits of papers, pager, mobile phone, any electronic device, etc. except the Admit Card inside the examination room/hall.
10. Rough work is to be done on the space provided for this purpose in the Test Booklet only. This space is given at the bottom of each page and in four pages (Page 20 – 23) at the end of the booklet.
11. On completion of the test, the candidate must hand over the Answer Sheet to the invigilator on duty in the Room/Hall. However, the candidates are allowed to take away this Test Booklet with them.
12. The CODE for this Booklet is A. Make sure that the CODE printed on Side – 2 of the Answer Sheet and also tally the serial number of the Test Booklet and Answer Sheet are the same as that on this booklet. In case of discrepancy, the candidate should immediately report the matter to the invigilator for replacement of both the Test Booklet and the Answer Sheet.
13. Do not fold or make any stray mark on the Answer Sheet.

JEE-MAIN-2017 SOLUTIONS

PHYSICS

1. A man grows into a giant such that his linear dimensions increase by a factor of 9. Assuming that his density remains same, the stress in the leg will change by a factor of :

- (1) 9 (2) $\frac{1}{9}$ (3) 81 (4) $\frac{1}{81}$

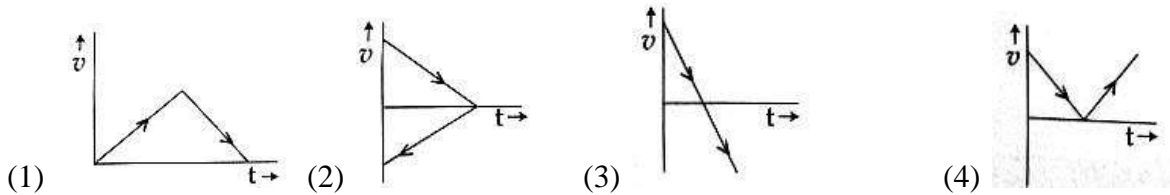
Key. (1)

Sol. $\text{Stress} = \frac{mg}{A} \times \frac{l}{l} = \frac{mgl}{v} = \rho gl$

$\therefore l$ becomes 9 times

\therefore stress becomes 9 times

2. A body is thrown vertically upwards. Which one of the following graphs correctly represent the velocity vs time ?



Key (3)

Sol. The slope i.e. acceleration must remain same.

3. A body of mass $m = 10^{-2}$ kg is moving in a medium and experiences a frictional force $F = -kv^2$. Its initial speed is $v_0 = 10 \text{ ms}^{-1}$. If, after 10s, its energy is $\frac{1}{8}mv_0^2$, the value of k will be :

- (1) $10^{-3} \text{ kg m}^{-1}$ (2) $10^{-3} \text{ kg s}^{-1}$
 (3) $10^{-4} \text{ kg m}^{-1}$ (4) $10^{-1} \text{ kg m}^{-1} \text{ s}^{-1}$

Key (3)

Sol. $F = -kv^2 = m \frac{dv}{dt} \Rightarrow \int_{v_0}^v v^{-2} dv = -\frac{k}{m} \int_0^{10} dt \Rightarrow \left[-\frac{1}{v} \right]_v^{v_0} = \frac{k}{m} \times 10 \Rightarrow \frac{10k}{m} = \left(\frac{1}{v} - \frac{1}{v_0} \right)$

$$\therefore \text{final K.E.} = \frac{1}{2} m \left(\frac{v_0}{2} \right)^2$$

$$\therefore \frac{10k}{m} \frac{1}{(v_0/2)} - \frac{1}{v_0} \Rightarrow \frac{10k}{m} = \frac{1}{v_0} \Rightarrow k = \frac{m}{10v_0} = 10^{-4} \text{kg m}^{-1}$$

4. A time dependent force $F = 6t$ acts on a particle of mass 1 kg. If the particle starts from rest, the work done by the force during the first 1 sec. will be :

- (1) 4.5 J (2) 22 J (3) 9 J (4) 18 J

Key (1)

Sol. $\Delta P = \int F \cdot dt = \int_0^1 6t \, dt \Rightarrow \Delta v = 6 \left[\frac{t^2}{2} \right]_0^1$

$$1 \times \Delta v = 6 \times \frac{1}{2}$$

$$\Delta v = 3 \text{m/s}$$

$$W = \Delta K = \frac{1}{2} m (v_f^2 - v_i^2)$$

$$\therefore v_i = 0, \Delta k = \frac{1}{2} \times 1 \times 9 = 4.5 \text{ J}$$

5. The moment of inertia of a uniform cylinder of length l and radius R about its perpendicular bisector is I . What is the ratio l/R such that the moment of inertia is minimum ?

- (1) $\frac{\sqrt{3}}{2}$ (2) $\frac{\sqrt{3}}{2}$ (3) 1 (4) $\frac{3}{\sqrt{2}}$

Key. (1)

$$I = \frac{ml^2}{12} + \frac{mR^2}{4}$$

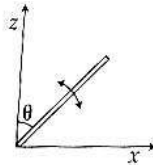
Sol. $I = \frac{m}{12} \left(\frac{m^2}{\pi^2 R^4 \rho^2} \right) + \frac{mR^2}{4}$

$$\frac{dI}{dR} = \frac{m^3}{12\pi^2 \rho^2} \left\{ \frac{d}{dR} (R^{-4}) \right\} + \frac{m}{4} \{2R\} = 0$$

$$\frac{m^3}{12\pi^2\rho^2}(4R^{-5}) = \frac{mR}{2}$$

$$\frac{l^2}{R^2} = \frac{3}{2} \Rightarrow \frac{l}{R} = \sqrt{\frac{3}{2}}$$

6. A cylinder uniform rod of mass M and length l is pivoted at one end so that it can rotate in a vertical plane (see figure). There is negligible friction at the pivot. The free end is held vertically above the pivot and then released. The angular acceleration of the rod when it makes an angle θ with the vertical is :



- (1) $\frac{3g}{2l} \sin \theta$ (2) $\frac{2g}{3l} \sin \theta$ (3) $\frac{3g}{2l} \cos \theta$ (4) $\frac{2g}{3l} \cos \theta$

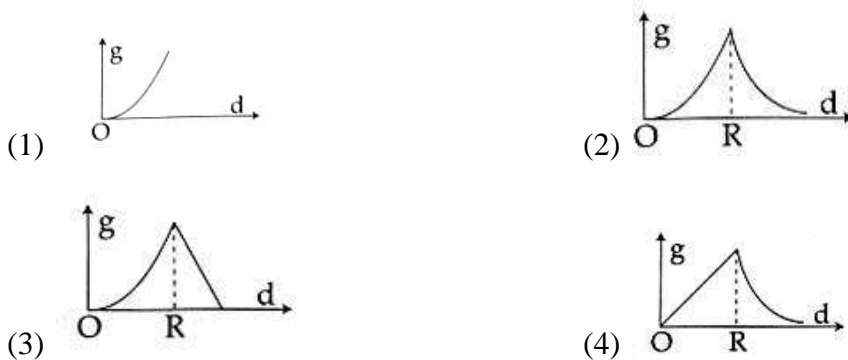
Key. (1)

Sol. $\tau = I\alpha$

$$mg \times \frac{l}{2} \sin \theta = \frac{ml^2}{3} \cdot \alpha$$

$$\alpha = \frac{3g}{2l} \sin \theta$$

7. The variation of acceleration due to gravity g with distance d from centre of the earth is best represented by (R = Earth's radius) :



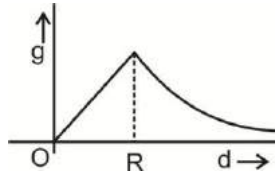
Key. (4)

Sol. $g = \frac{GM}{R^3}d$ when $d < R$

i.e. $g \propto d$

$g = \frac{GM}{d^2}$ when $d > R$

i.e. $g \propto \frac{1}{d^2}$



8. A copper ball of mass 100 gm is at a temperature T . It is dropped in a copper calorimeter of mass 100 gm, filled with 170 gm of water at room temperature. Subsequently, the temperature of the system is found to be 75°C . T is given by :

(Given : room temperature = 30°C , specific heat of copper = $0.1 \text{ cal/gm}^\circ\text{C}$)

- (1) 800°C (2) 885°C (3) 1250°C (4) 825°C

Key. (2)

Sol. heat given = heat taken

$$100 \times 0.1 \times (T - 75^\circ) = (100 \times 0.1 \times 45^\circ) + (170 \times 1 \times 45^\circ)$$

$$\Rightarrow (T - 75^\circ)10 = 450 + 7650$$

$$\Rightarrow T = 810 + 75 = 885^\circ\text{C}$$

9. An external pressure P is applied on a cube at 0°C so that it is equally compressed from all sides. K is the bulk modulus of the material of the cube and α is its coefficient of linear expansion. Suppose we want to bring the cube to its original size by heating. The temperature should be raised by :

- (1) $\frac{P}{3\alpha K}$ (2) $\frac{P}{\alpha K}$ (3) $\frac{3\alpha}{PK}$ (4) $3PK\alpha$

Key. (1)

Sol. $K = \frac{P}{\Delta V/V} \Rightarrow \frac{\Delta V}{V} = \frac{P}{K} = \gamma \Delta T = 3\alpha \Delta T$

$$\Delta T = \frac{P}{3\alpha K}$$

10. C_p and C_v are specific heats at constant pressure and constant volume respectively. It is observed that

$$C_p - C_v = a \text{ for hydrogen gas}$$

$$C_p - C_v = b \text{ for nitrogen gas}$$

The correct relation between a and b is :

$$(1) a = \frac{1}{14} b$$

$$(2) a = b$$

$$(3) a = 14 b$$

$$(4) a = 28 b$$

Key. (3)

Sol. If the given specific heats C_p and C_v are molar specific heats then the answer will be (2).

Otherwise

$$a = C_p - C_v = \frac{R}{M} = \frac{R}{2}$$

$$b = C_p - C_v = \frac{R}{28}$$

$$a = \frac{R}{2}$$

$$b = \frac{R}{2} \times \frac{1}{14}, \quad b = \frac{a}{14}$$

11. The temperature of an open room of volume 30 m^3 increases from 17°C to 27°C due to the sunshine. The atmospheric pressure in the room remains $1 \times 10^5 \text{ Pa}$. If n_i and n_f are the number of molecules in the room before and after heating, then $n_f - n_i$ will be

$$(1) -1.61 \times 10^{23}$$

$$(2) 1.38 \times 10^{23}$$

$$(3) 2.5 \times 10^{25}$$

$$(4) -2.5 \times 10^{25}$$

Key (4)

Sol. $PV = \frac{n}{N_A} \cdot RT$ (n = number of molecules)

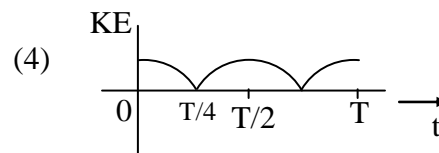
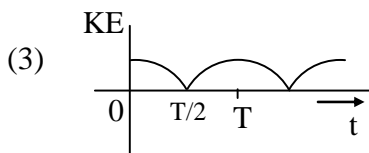
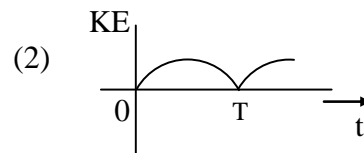
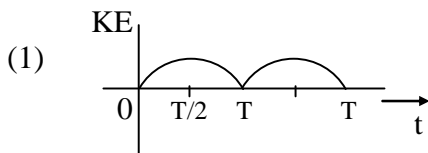
$$n_i = \frac{10^5 \times 30}{8.314 \times 290} N_A$$

$$n_f = \frac{10^5 \times 30}{8.314 \times 300} \times 6.022 \times 10^{23}$$

$$n_f - n_i = \frac{10^5 \times 30}{8.314} \left(\frac{1}{290} - \frac{1}{300} \right) N_A$$

$$n_f - n_i = -2.5 \times 10^{25}$$

12. A particle is executing simple harmonic motion with a time period T. At time t = 0, it is at its position of equilibrium. The kinetic energy – time graph of the particle will look like



Key (4)

Sol. If the oscillation frequency is f then frequency of oscillation of kinetic energy will be 2f.

So the period of oscillation of kinetic energy will be T/2. Also since at t = 0, particle is at mean position so the KE is maximum at t = 0.

∴ (4) is correct.

13. An observer is moving with half the speed of light towards a stationary microwave source emitting waves at frequency 10 GHz. What is the frequency of the microwave measured by the observer? (speed of light = 3×10^8 m/s)

- (1) 10.1 GHz (2) 12.1 GHz (3) 17.3 GHz (4) 15.3 GHz

Key (3)

Sol. $f_{\text{source}} = 10 \text{ GHz}$

For Doppler effect in EM waves,

$$\frac{\Delta f}{f} = \sqrt{\frac{1+v/c}{1-v/c}} - 1$$

Here, $v = c/2$ (v = velocity of observer)

$$\Rightarrow \frac{\Delta f}{f} = \sqrt{\frac{1+c/2c}{1-c/2c}} - 1 = \sqrt{\frac{3}{2}} - 1 = \sqrt{3} - 1$$

$$\frac{f_{\text{apparent}} - f_{\text{source}}}{f_{\text{source}}} = \sqrt{3} - 1$$

$$\frac{f_{\text{apparent}}}{f_{\text{source}}} = \sqrt{3} \Rightarrow f_{\text{apparent}} = \sqrt{3} \times 10 = 17.3 \text{ GHz}$$

14. An electric dipole has a fixed dipole moment \vec{p} , which makes angle θ with respect to x-axis. When subjected to an electric field $\vec{E}_1 = E\hat{i}$, it experiences a torque $\vec{T}_1 = \tau\hat{k}$. When subjected to another electric field $\vec{E}_2 = \sqrt{3}E\hat{j}$ it experiences a torque $\vec{T}_2 = -\vec{T}_1$. The angle θ is

- (1) 30° (2) 45° (3) 60° (4) 90°

Key (3)

Sol Here $\vec{P} = P\cos\theta\hat{i} + P\sin\theta\hat{j}$

$$\vec{E}_1 = E\hat{i}$$

$$\therefore \vec{\tau}_1 = \vec{P} \times \vec{E}_1 = PE\sin\theta(-\hat{k})$$

$$\text{Given, } \vec{E}_2 = \sqrt{3}E\hat{j} = \sqrt{3}E\hat{j}$$

$$\therefore \vec{\tau}_2 = \vec{P} \times \vec{E}_2 = \sqrt{3}PE\cos\theta\hat{k}$$

$$\text{Given } \vec{\tau}_2 = -\vec{\tau}_1$$

$$\therefore \sqrt{3}PE\cos\theta = PE\sin\theta$$

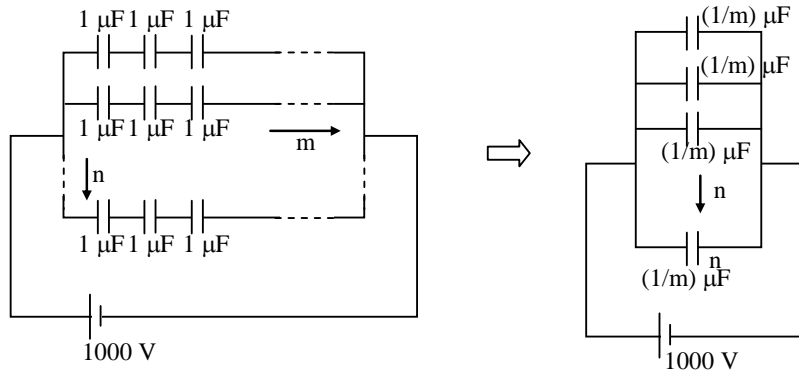
$$\therefore \tan\theta = \sqrt{3} \text{ or } \theta = 60^\circ$$

$$\therefore (3)$$

15. A capacitance of $2 \mu\text{F}$ is required in an electrical circuit across a potential difference of 1.0 kV . A large number of $1 \mu\text{F}$ capacitors are available which can withstand a potential difference of not more than 300 V . The minimum number of capacitors required to achieve this is

- (1) 2 (2) 16 (3) 24 (4) 32

Key (4)



Sol. Equivalent capacitance $C_{eq} = 1\mu\text{F} \left(\frac{n}{m} \right) = 2\mu\text{F}$

$$\frac{n}{m} = 2$$

Potential difference each capacitance = $1000/m$

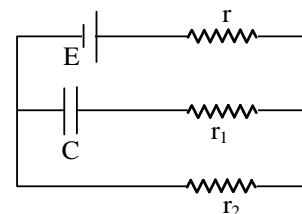
$$V_{\max} = 300 \text{ V} \Rightarrow 300 > (1000/m)$$

$$\Rightarrow m > (10/3) \Rightarrow m = 4$$

$$\text{And } (n/m) = 2 \Rightarrow n = 8$$

$$\text{Total number of capacitors} = m \times n = 8 \times 4 = 32$$

16. In the given circuit diagram when the current reaches steady state in the circuit, the charge on the capacitor of capacitance C will be



- (1) CE (2) $CE \frac{r_1}{(r_2 + r)}$

(3) $CE \frac{r_2}{(r+r_2)}$ (4) $CE \frac{r_1}{(r_1+r)}$

Key (3)

Sol. In steady state there will be no current through capacitor and the current through E, r and r_2 will be $i = \frac{E}{r+r_2}$.

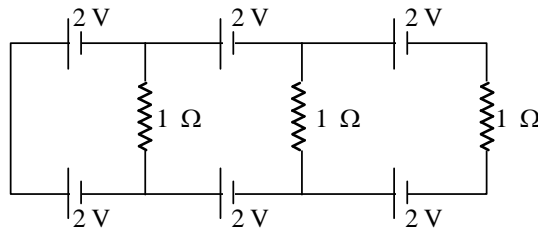
Also the potential difference across C will be same as that of r_2 .

$$\therefore V_C = ir_2 = \frac{Er_2}{r+r_2}$$

So the charge stored in C, $q = CV_C = \frac{ECr_2}{r+r_2}$

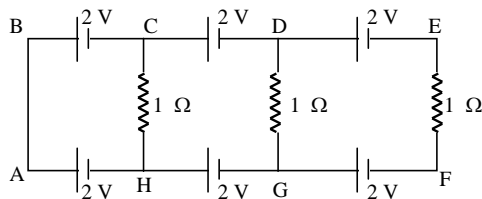
\therefore (3) is correct.

17. In the circuit, the current in each resistance is



- (1) 1A (2) 0.25 A (3) 0.5 A (4) 0 A

Key (4)



Sol. If we assume the potential at A to be 2V

Then, $V_B = V_A = 2\text{ V}$

Then $V_C = V_H = 0\text{ V}$

$\therefore V_C - V_H = 0$ i.e. $i_{CH} = 0$

Also $V_D = V_G = -2\text{ V}$ $\therefore V_D - V_G = 0$, $\therefore i_{DG} = 0$

And $V_E = V_F = -4V$

$$\therefore V_E - V_F = 0 \quad \therefore i_{EF} = 0$$

So current in all the resistors is zero.

$$\therefore (4)$$

18. A magnetic needle of magnetic moment $6.7 \times 10^{-2} \text{ Am}^2$ and moment of inertia $7.5 \times 10^{-6} \text{ kg m}^2$ is performing simple harmonic oscillations in a magnetic field of 0.01 T. Time taken for 10 complete oscillations is

- (1) 6.65 s (2) 8.89 s (3) 6.98 s (4) 8.76 s

Key (1)

Sol.
$$T = 2\pi \sqrt{\frac{I}{MB}} = 2\pi \sqrt{\frac{7.5 \times 10^{-6}}{6.7 \times 10^{-2} \times 0.01}}$$

$$T = 0.665 \text{ s}$$

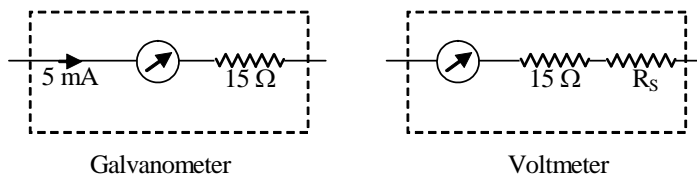
So for 10 oscillations time taken will be $10T = 6.65 \text{ s}$.

19. When a current of 5 mA is passed through a galvanometer having a coil of resistance 15Ω , it shows full scale deflection. The value of the resistance to be put in series with the galvanometer to convert it into a voltmeter of range 0 – 10 V is

- (1) $1.985 \times 10^3 \Omega$ (2) $2.045 \times 10^3 \Omega$ (3) $2.535 \times 10^3 \Omega$ (4) $4.005 \times 10^3 \Omega$

Key (1)

Sol. Full scale deflection (FSD) of current = 5 mA

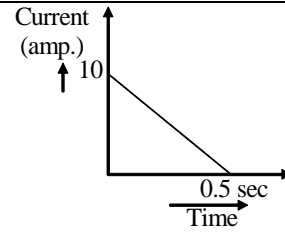


FSD voltage = 10 V

$$\Rightarrow V_{\max} = i(R_s + R_G) \Rightarrow 10 = 5 \times 10^{-3} (R_s + 15)$$

$$2000 = R_s + 15 \Rightarrow R_s = 1985 \Omega$$

20. In a coil of resistance 100Ω , a current is induced by changing the magnetic flux through it as shown in the figure. The magnitude of change in flux through the coil is



- (1) 200 Wb (2) 225 Wb
 (3) 250 Wb (4) 275 Wb

Key (3)

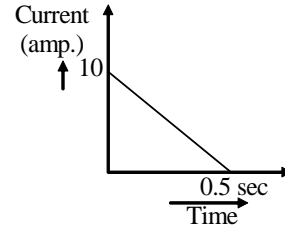
Sol Induced emf = $-\frac{d\phi}{dt}$

$$i = \frac{\varepsilon}{R} \Rightarrow i = -\frac{1}{R} \frac{d\phi}{dt} \Rightarrow d\phi = -iRdt$$

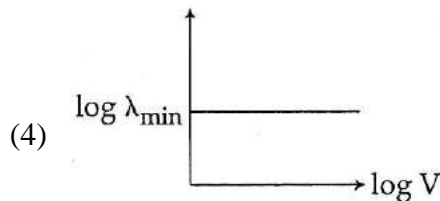
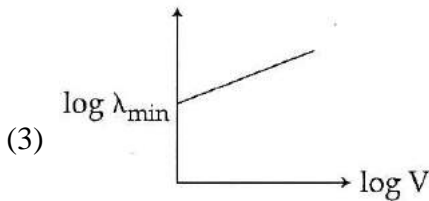
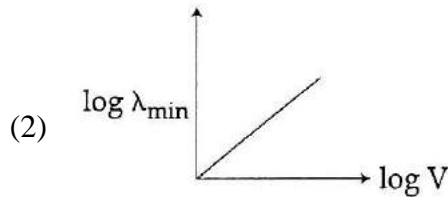
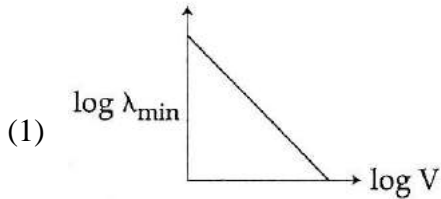
$$\Delta\phi = -\int iRdt = -R \int idt \quad (\int idt \text{ area under } i\text{-}t \text{ graph})$$

Magnitude of $\Delta\phi = R \times \text{Area}$

$$= \frac{100 \times 10 \times 0.5}{2} = 250 \text{ Wb}$$



21. An electron beam is accelerated by a potential difference V to hit a metallic target to produce X-rays. It produces continuous as well as characteristic X-rays. If λ_{\min} is the smallest possible wavelength of X-ray in the spectrum, the variation of $\log \lambda_{\min}$ with $\log V$ is correctly represented in

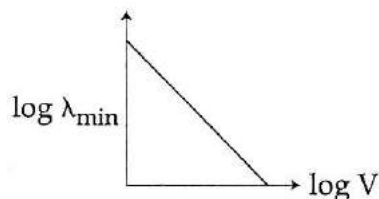


Key (1)

Sol. $\lambda_{\min} = \frac{hc}{eV}$

$$\log \lambda_{\min} = \log \frac{hc}{e} - \log V$$

It is a straight line with -ve slope and +ve intercept



22. A diverging lens with magnitude of focal length 25 cm is placed at a distance of 15 cm from a converging lens of magnitude of focal length 20 cm. A beam of parallel light falls on the diverging lens. The final image formed is

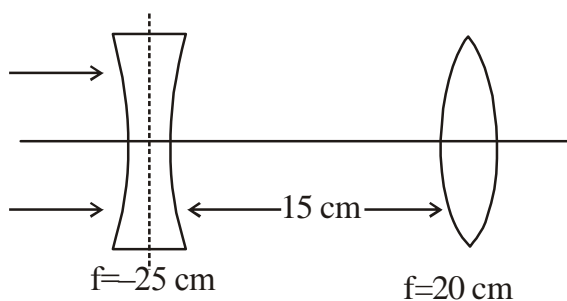
- (1) real and at a distance of 40 cm from convergent lens
- (2) virtual and at a distance of 40 cm from convergent lens
- (3) real and at a distance of 40 cm from the divergent lens
- (4) real and at a distance of 6 cm from the convergent lens.

Key (1)

Sol. Image formed by first lens $v = -25$

For converging lens

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$



$$\Rightarrow \frac{1}{v} - \frac{1}{-40} = \frac{1}{20} \Rightarrow \frac{1}{v} = \frac{1}{20} - \frac{1}{40} = \frac{1}{40}$$

$$v = 40$$

23. In a Young's double slit experiment, slits are separated by 0.5 mm, and the screen is placed 150 cm away. A beam of light consisting of two wavelengths, 650 nm and 520 nm, is used to obtain interference fringes on the screen. The least distance from the common central maximum to the point where the bright fringes due to both the wavelengths coincide is

- (1) 1.56 mm (2) 7.8 mm (3) 9.75 mm (4) 15.6 mm

Key (2)

Sol. $d = 0.5 \times 10^{-3}$; $D = 1.5$ m, $\lambda_1 = 650 \times 10^{-9}$; $\lambda_2 = 520 \times 10^{-9}$

$$\frac{m\lambda_1 D}{d} = \frac{n\lambda_2 D}{d}$$

$$m \times 650 \times 10^{-9} = n \times 520 \times 10^{-9}$$

$$5m = 4n \Rightarrow m = 4; n = 5$$

$$y = \frac{mD\lambda_1}{d} = \frac{4 \times 650 \times 10^{-9} \times 1.5}{0.5 \times 10^{-3}} = 12 \times 650 \times 10^{-6}$$

$$= 7800 \times 10^{-6} = 7.8 \times 10^{-3} \text{ m} = 7.8 \text{ mm}$$

24. A particle A of mass m and initial velocity v collides with a particle B of mass $\frac{m}{2}$ which is at rest.

The collision is head on, and elastic. The ratio of the de-Broglie wavelengths λ_A to λ_B after the collision is :

- (1) $\frac{\lambda_A}{\lambda_B} = \frac{1}{3}$ (2) $\frac{\lambda_A}{\lambda_B} = 2$ (3) $\frac{\lambda_A}{\lambda_B} = \frac{2}{3}$ (4) $\frac{\lambda_A}{\lambda_B} = \frac{1}{2}$

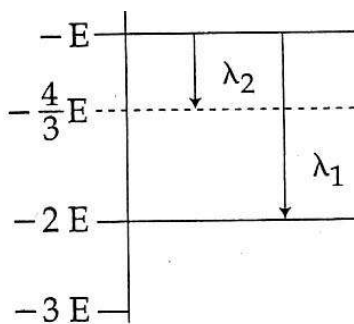
Key (2)

Sol. Using C.O.M.

$$v_A = \frac{m/2}{3m/2} v = \frac{v}{3}; \quad v_B = \frac{2m}{3m/2} v = \frac{4}{3} v$$

$$\lambda = \frac{h}{mv} \Rightarrow \frac{\lambda_A}{\lambda_B} = \frac{\frac{m}{2} \times \frac{4v}{3}}{m \times \frac{v}{3}} = 2:1$$

25. Some energy levels of a molecule are shown in the figure. The ratio of the wavelengths $r = \lambda_1 / \lambda_2$, is given by



- (1) $r = \frac{4}{3}$ (2) $r = \frac{2}{3}$ (3) $r = \frac{3}{4}$ (4) $r = \frac{1}{3}$

Key (4)

Sol. According to formula $\Delta E = \frac{hc}{\lambda}$

$$\text{So } \left\{ -E - \left(-\frac{4E}{3} \right) \right\} = \frac{hc}{\lambda_2}$$

$$\frac{hc}{\lambda_2} = \frac{E}{3} \quad \dots(1)$$

Similarly $\frac{hc}{\lambda_1} = E$

$$\text{So, } \frac{\lambda_1}{\lambda_2} = \frac{1}{3} = r$$

So correct option is (4)

26. A radioactive nucleus A with a half life T, decays into a nucleus B. At $t = 0$, there is no nucleus B. At sometime t, the ratio of the number of B to that of A is 0.3. Then, t is given by

- (1) $t = \frac{T \log 2}{2 \log 1.3}$ (2) $t = T \frac{\log 1.3}{\log 2}$ (3) $t = T \log(1.3)$ (4) $t = \frac{T}{\log(1.3)}$

Key (2)

Sol. the element formation for B

$$\text{For half life } t_{1/2} = T = \frac{\log(2)}{\lambda}$$

$$\text{At } t = 0 ; N_A = N_0$$

$$\text{So } N_B = N_0 (1 - e^{-\lambda t})$$

$$\text{So, } \frac{N_B}{N_A} = \frac{(1 - e^{-\lambda t})}{(e^{-\lambda t})}$$

$$0.3 = e^{\lambda t} - 1$$

$$1.3 = e^{\lambda t}$$

$$t = \frac{\log(1.3)}{\lambda} = T \frac{\log(1.3)}{\log(2)}$$

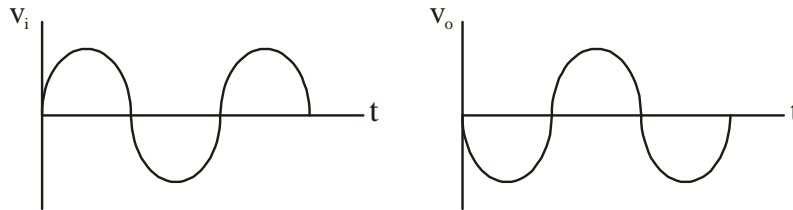
So option is (2)

27. In a common emitter amplifier circuit using an n-p-n transistor, the phase difference between the input and the output voltages will be

- (1) 45° (2) 90° (3) 135° (4) 180°

Key (4)

Sol. 180°



28. In amplitude modulation, sinusoidal carrier frequency used is denoted by ω_c and the signal frequency is denoted by ω_m . The bandwidth ($\Delta\omega_m$) of the signal is such that $\Delta\omega_m \ll \omega_c$. Which of the following frequencies is not contained in the modulated wave?

- (1) ω_m (2) ω_c (3) $\omega_m + \omega_c$ (4) $\omega_c - \omega_m$

Key (1)

Sol. Equation of amplitude modulated wave is

$$\cos(t) = A_c \sin \omega_c t + \frac{\mu A_c}{2} \cos(\omega_c - \omega_m)t + \frac{\mu A_c}{2} \cos(\omega_c + \omega_m)t$$

ω_m is not contained in this.

29. Which of the following statements is false?

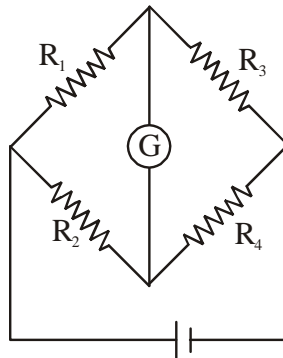
- (1) Wheatstone bridge is the most sensitive when all the four resistances are of the same order of magnitude
- (2) In a balanced wheatstone bridge if the cell and the galvanometer are exchanged, the null point is disturbed
- (3) A rheostat can be used as a potential divider
- (4) Kirchoff's second law represents energy conservation

Key (2)

Sol. In a balanced Wheatstone bridge if the cell and the galvanometer one exchanged the position of null point remain same. Hence option is 2

For Wheatstone bridge

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$



Null point is not disturbed in galvanometer and cell are interchanged

30. The following observations were taken for determining surface tension T of water by capillary method

Diameter of capillary, $D = 1.25 \times 10^{-2}$ m

Rise of water, $h = 1.45 \times 10^{-2}$ m

Using $g = 9.80 \text{ m/s}^2$ and the simplified relation $T = \frac{rgh}{2} \times 10^3 \text{ N/m}$, the possible error in surface tension is closest to

- (1) 0.15%
- (2) 1.5%
- (3) 2.4%
- (4) 10%

Key (2)

Sol. $T = \frac{rgh}{2} = \frac{dgh}{4}$

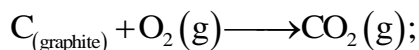
$$\frac{\Delta T}{T} = \frac{\Delta d}{d} + \frac{\Delta g}{g} + \frac{\Delta h}{h}$$

$$\frac{\Delta T}{T} = \left(\frac{1}{125} + \frac{1}{145} + \frac{1}{980} \right)$$

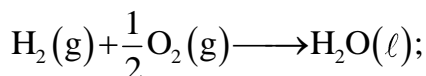
$$\frac{\Delta T}{T} \times 100 = 1.5\%$$

CHEMISTRY

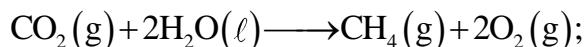
31. Given



$$\Delta_r H^\circ = -393.5 \text{ kJ mol}^{-1}$$

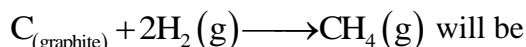


$$\Delta_r H^\circ = -285.8 \text{ kJ mol}^{-1};$$



$$\Delta_r H^\circ = +890.3 \text{ kJ mol}^{-1}$$

Based on the above thermochemical equations, the value of $\Delta_r H^\circ$ at 298 K for the reaction



(1) $-74.8 \text{ kJ mol}^{-1}$ (2) $-144.0 \text{ kJ mol}^{-1}$ (3) $+74.8 \text{ kJ mol}^{-1}$ (4) $+144.0 \text{ kJ mol}^{-1}$

Key. (1)

$$\text{Sol. } \Delta H_r = \Delta H_{\text{comb.}}(\text{Graphite}) + 2\Delta H_{\text{comb.}}(H_2) - \Delta H_{\text{comb.}}(CH_4)$$

$$= (-393.5) + 2 \times (-285.8) - (-890.3) \text{ kJ}$$

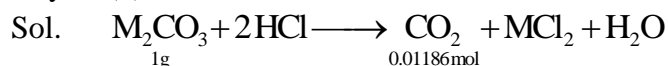
$$= 890.3 - 965.1$$

$$= -74.8 \text{ kJ mol}^{-1}$$

32. 1 gram of a carbonate (M_2CO_3) on treatment with excess HCl produces 0.01186 of CO_2 . The molar mass of M_2CO_3 in g mol^{-1} is

(1) 118.6 (2) 11.86 (3) 1186 (4) 84.3

Key. (4)



Since 1 mole M_2CO_3 releases 1 mole of CO_2 on treatment with acid so,

$$0.01186 \text{ mole of } M_2CO_3 = \frac{1}{0.01186} = 84.3 \text{ g mol}^{-1}$$

33. ΔU is equal to

(1) Adiabatic work (2) Isothermal work (3) Isochoric work (4) Isobaric work

Key. (1)

$$\text{Sol. } \Delta U = q + W$$

$$\text{When } q = 0, \Delta U = W$$

34. The Tyndall is observed only when following conditions are satisfied :

(a) The diameter of the dispersed particles is much smaller than the wavelength of the light used.

(b) The diameter of the dispersed particle is not much smaller than the wavelength of the light used.

(c) The refractive indices of the dispersed phase and dispersion medium are almost similar in magnitude.

(d) The refractive indices of the dispersed phase and dispersion medium differ greatly in magnitude.

(1) (a) and (c) (2) (b) and (c) (3) (a) and (d) (4) (b) and (d)

Key.

(4)

Sol. Conditions for Tyndall effect are

(b) The diameter of the dispersed particle is not much smaller than the wavelength of the light used.

(d) The refractive indices of the dispersed phase and the dispersion medium differ greatly in magnitude

35. A metal crystallizes in a face centred cubic structure. If the edge length of its unit cell is 'a', the closest approach between two atoms in metallic crystal will be

(1) $\sqrt{2}a$ (2) $\frac{a}{\sqrt{2}}$ (3) $2a$ (4) $2\sqrt{2}a$

Key. (2)

Sol. Closed approach in FCC = $\frac{\text{face diagonal}}{2} = \frac{\sqrt{2}a}{2} = \frac{a}{\sqrt{2}}$

36. Given

$$E_{\text{Cl}_2/\text{Cl}^-}^{\circ} = 1.36 \text{ V}, E_{\text{Cr}^{3+}/\text{Cr}}^{\circ} = -0.74 \text{ V}$$

$E_{\text{Cr}_2\text{O}_7^{2-}/\text{Cr}^{3+}}^{\circ} = 1.33 \text{ V}$, $E_{\text{MnO}_4^-/\text{Mn}^{2+}}^{\circ} = 1.51 \text{ V}$. Among the following, the strongest reducing agent is

(1) Cr^{3+} (2) Cl^- (3) Cr (4) Mn^{2+}

Key. (3)

Sol. Cr^{3+}/Cr has least reduction potential which implies that 'Cr' has maximum tendency to get oxidized.

37. The freezing point of benzene decreases by 0.45°C when 0.2 g of acetic acid is added to 20 g of benzene. If acetic acid associates to form a dimer in benzene, percentage association of acetic acid in benzene will be

(K_f for benzene = $5.12 \text{ K kg mol}^{-1}$)

(1) 74.6% (2) 94.6% (3) 64.6% (4) 80.4%

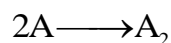
Key. (2)

Sol. $\Delta T_f = i K_f \cdot m$

$$0.45 = i \cdot 5.12 \cdot \frac{0.2/60}{20/1000}$$

$$0.45 = i \times 5.12 \times \frac{1}{300} \times \frac{1000}{20}$$

$$i = \frac{6 \times 0.45}{5.12} = 0.5273 \approx 0.53$$



$$1-x = \frac{x}{2}$$

$$i = 1 - \frac{x}{2} = 0.53$$

$$\frac{x}{2} = 1 - 0.53 = 0.47$$

$$x = 0.94$$

94% associated

38. The radius of the second Bohr orbit for hydrogen atom is :
 (Planck's Const. $h = 6.6262 \times 10^{-34}$ Js; mass of electron = 9.1091×10^{-31} kg; charge of electron $e = 1.60210 \times 10^{-19}$ C; permittivity of vacuum $\epsilon_0 = 8.854185 \times 10^{-12}$ kg⁻¹ m⁻³ A²)
 (1) 0.529 Å^o (2) 2.12 Å^o (3) 1.65 Å^o (4) 4.76 Å^o

Key. (2)

Sol. $r_n = \frac{0.529 \times n^2}{Z} = 0.529 \times 4 = 2.116 = 2.12 \text{ \AA}$

39. Two reactions R₁ and R₂ have identical pre-exponential factors. Activation energy of R₁ exceeds that of R₂ by 10 kJ mol⁻¹. If k₁ and k₂ are rate constants for reactions R₁ and R₂ respectively at 300 K, then in (k₂/k₁) is equal to
 (1) 6 (2) 4 (3) 8 (4) 12

Key. (2)

Sol. $K_1 = A e^{-E_{a1}/RT}$

$$K_2 = A e^{-E_{a2}/RT}$$

$$\frac{K_2}{K_1} = e^{-\frac{1}{RT}(E_{a2}-E_{a1})}$$

$$\ln \frac{K_2}{K_1} = -\frac{1}{RT} \times -10,000$$

$$= \frac{10000}{8.31 \times 300} = 4$$

40. pK_a of a weak acid (HA) and pK_b of a weak base (BOH) are 3.2 and 3.4, respectively. The pH of their salt (AB) solution is
 (1) 7.0 (2) 1.0 (3) 7.2 (4) 6.9

Key. (4)

Sol. $\text{pH} = 7 + \frac{1}{2} \text{pKa} - \frac{1}{2} \text{pKb}$
 $= 7 + 1.6 - 1.7$
 $= 6.9$

41. Both lithium and magnesium display several similar properties due to the diagonal relationship, however, the one which is incorrect, is
 (1) both form nitrides
 (2) nitrates of both Li and Mg yield NO_2 and O_2 on heating
 (3) both form basic carbonates
 (4) both form soluble bicarbonates

41. (3)

Sol. Mg forms basic carbonate that is $\text{MgCO}_3 \cdot \text{Mg(OH)}_2$ while Li does not form basic carbonate.

42. Which of the following species is not paramagnetic?

(1) O_2 (2) B_2 (3) NO (4) CO

42. (4)

Sol. $\text{B}_2 = 10 = \sigma 1s^2 \sigma^* 1s^2 \sigma 2s^2 \sigma^* 2s^2 \pi 2p_x^1 = \pi 2p_x^1$ (paramagnetic)

$\text{O}_2 = 16 = \sigma 1s^2 \sigma^* 1s^2 2s^2 \sigma^* 2s^2 \pi 2p_x^2 = \pi 2p_x^2 \sigma^* 2p_z^2 \pi 2p_x^{*1} = \pi 2p_x^{*1}$ (paramagnetic)

$\text{NO} = 15 = \sigma 1s^2 \sigma^* 1s^2 2s^2 \sigma^* 2s^2 \pi 2p_x^2 = \pi 2p_x^2 \sigma^* 2p_z^2 \pi 2p_x^{*1} = \pi^* 2p_x$ (paramagnetic)

$\text{CO} = 14 = \sigma 1s^2 \sigma^* 1s^2 2s^2 \sigma^* 2s^2 \pi 2p_x^2 = \pi 2p_x^2 \sigma^* 2p_z^2$ (Diamagnetic)

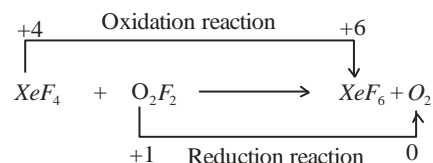
43. Which of the following reactions is an example of a redox reaction?

(1) $\text{XeF}_6 + \text{H}_2\text{O} \rightarrow \text{XeOF}_4 + 2\text{HF}$ (2) $\text{XeF}_6 + 2\text{H}_2\text{O} \rightarrow \text{XeO}_2\text{F}_2 + 4\text{HF}$

(3) $\text{XeF}_4 + \text{O}_2\text{F}_2 \rightarrow \text{XeF}_6 + \text{O}_2$ (4) $\text{XeF}_2 + \text{PF}_5 \rightarrow [\text{XeF}]^+ \text{PF}_6^-$

43. (3)

Sol.



44. A water sample has ppm level concentration of following anions

$\text{F}^- = 10$; $\text{SO}_4^{2-} = 100$; $\text{NO}_3^- = 50$

The anion/anions that make/makes the water sample unsuitable for drinking is/are

(1) only F^- (2) only SO_4^{2-}

(3) only NO_3^- (4) Both SO_4^{2-} and NO_3^-

44. (1)

Sol. F^- ion concentration above 2 ppm causes brown mottling of teeth. At the same time excess fluoride (over 10 ppm) causes harmful effect to bones and teeth.

45. The group having isoelectronic species is

(1) $\text{O}^{2-}, \text{F}^-, \text{Na}, \text{Mg}^{2+}$ (2) $\text{O}^-, \text{F}^-, \text{Na}^+, \text{Mg}^{2+}$

(3) $\text{O}^{2-}, \text{F}^-, \text{Na}^+, \text{Mg}^{2+}$ (4) $\text{O}^-, \text{F}^-, \text{Na}, \text{Mg}^+$

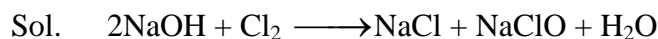
45. (3)

Sol. $\text{O}^{2-}, \text{F}^-, \text{Na}^+$ and Mg^{2+} have 10 electrons in each hence all these are isoelectronic species.

46. The products obtained when chlorine gas reacts with cold and dilute aqueous NaOH are

- (1) Cl^- and ClO^- (2) Cl^- and ClO_2^-
 (3) ClO^- and ClO_3^- (4) ClO_2^- and ClO_3^-

46. (1)



47. In the following reactions, ZnO is respectively acting a/an

- (a) $\text{ZnO} + \text{Na}_2\text{O} \rightarrow \text{Na}_2\text{ZnO}_2$ (b) $\text{ZnO} + \text{CO}_2 \rightarrow \text{ZnCO}_3$
 (1) acid and acid (2) acid and base (3) base and acid (4) base and base

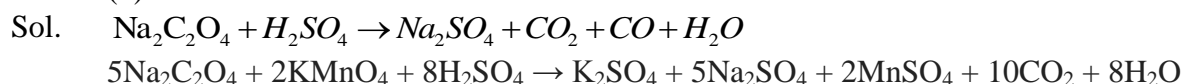
47. (2)

Sol. Na_2O is basic oxide hence in reaction (a) ZnO acts as an acid
 CO_2 is acidic oxide hence ZnO acts a base in reaction (b)

48. Sodium salt of an organic acid 'X' produces effervescence with conc. H_2SO_4 . 'X' reacts with the acidified aqueous CaCl_2 solution to give a white precipitate which decolourises acidic solution of KMnO_4 . 'X' is

- (1) CH_3COONa (2) $\text{Na}_2\text{C}_2\text{O}_4$ (3) $\text{C}_6\text{H}_5\text{COONa}$ (4) HCOONa

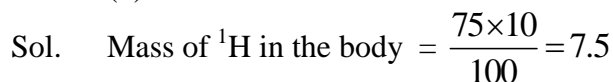
48. (2)



49. The most abundant elements by mass in the body of a healthy human adult are : Oxygen (61.4%); Carbon (22.9%), Hydrogen (10.0%); and Nitrogen (2.6%).The weight which a 75 kg person would gain it all ^1H atoms are replaced by ^2H atoms is

- (1) 7.5 g (2) 10 kg (3) 15 kg (4) 37.5 kg

49. (1)



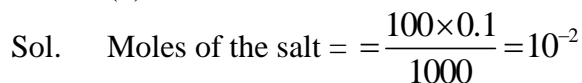
If ^1H is replace by ^2H then mass of $^2\text{H} = 15$ kg

Weight gain = 7.5 kg

50. On treatment of 100 mL of 0.1 M solution of $\text{CoCl}_3 \cdot 6\text{H}_2\text{O}$ with excess AgNO_3 ; 1.2×10^{22} ions are precipitated. The complex is

- (1) $[\text{Co}(\text{H}_2\text{O})_6]\text{Cl}_3$ (2) $[\text{Co}(\text{H}_2\text{O})_5\text{Cl}]\text{Cl}_2 \cdot \text{H}_2\text{O}$
 (3) $[\text{Co}(\text{H}_2\text{O})_4\text{Cl}_2]\text{Cl} \cdot 2\text{H}_2\text{O}$ (4) $[\text{Co}(\text{H}_2\text{O})_3\text{Cl}_3] \cdot 3\text{H}_2\text{O}$

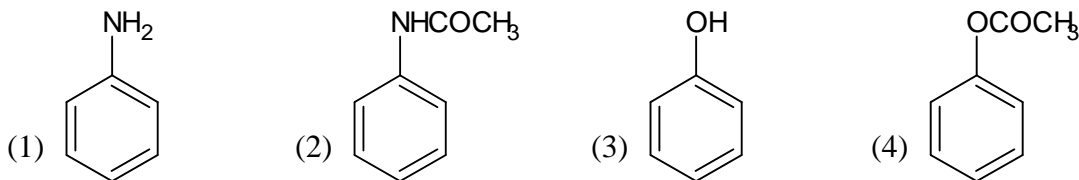
50. (2)



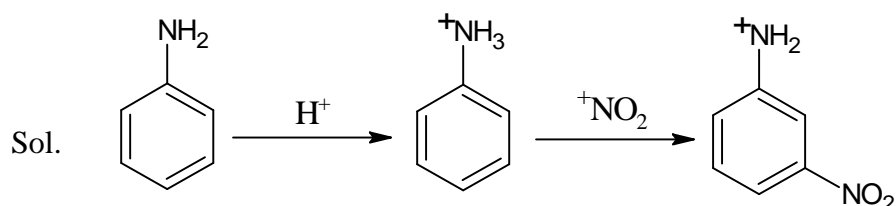
Moles of chloride ion from one mole of the salt = $\frac{1.2 \times 10^{22}}{6 \times 10^{23} \times 10^{-2}} = 2$

Hence two moles of chloride ions are present outside the coordination sphere.

51. Which of the following compounds will form significant amount of *meta* product during mono-nitration reaction?



Key (1)



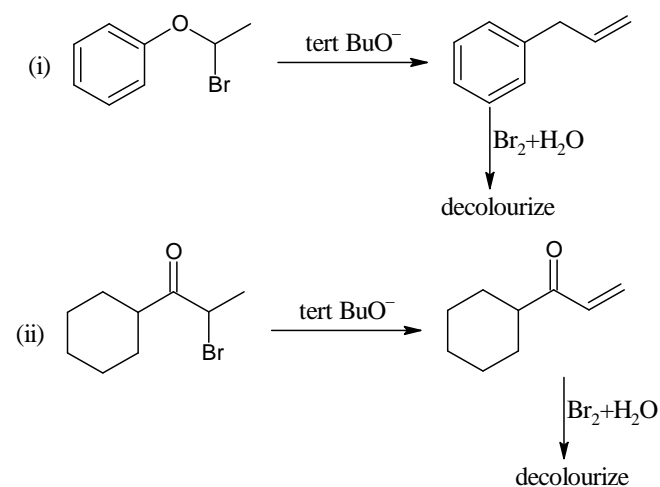
$-\text{NH}_3^{\oplus}$ is deactivating group

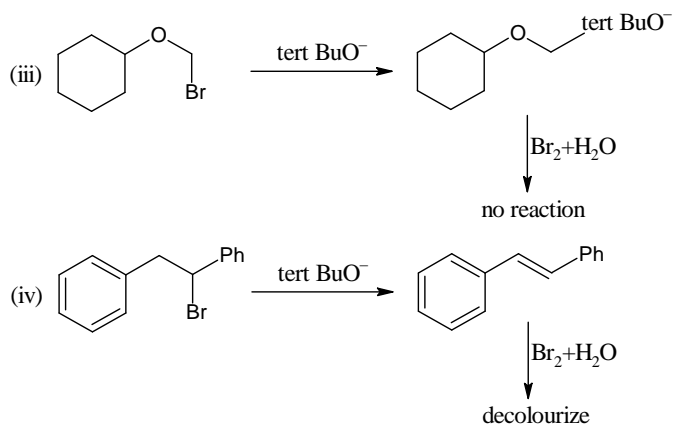
52. Which of the following, upon treatment with *tert*-BuONa followed by addition of bromine water, fails to decolourize the colour of bromine?



Key (3)

Sol.

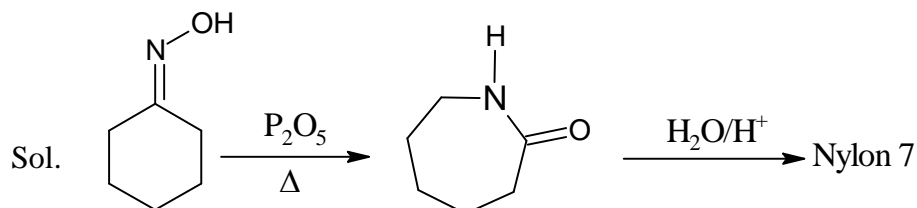




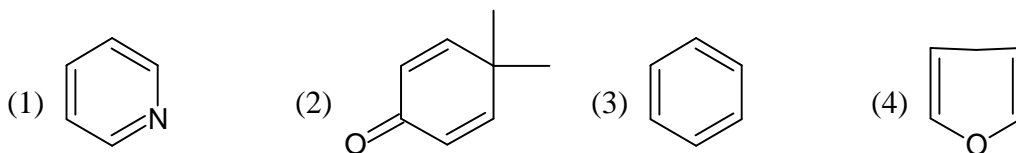
53. The formation of which of the following polymers involves hydrolysis reaction?

- (1) Nylon 6, 6 (2) Terylene (3) Nylon 6 (4) Bakelite

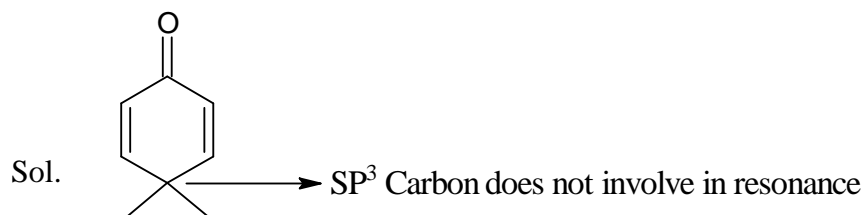
Key (3)



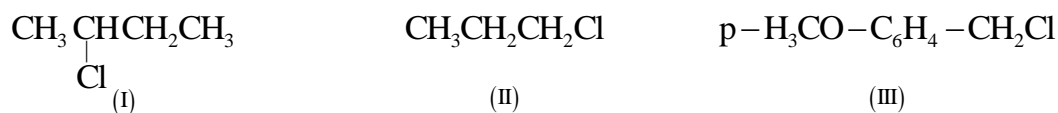
54. Which of the following molecules is least resonance stabilized?



Key (3)



55. The increasing order of the reactivity of the following halides for the S_N1 reaction is:

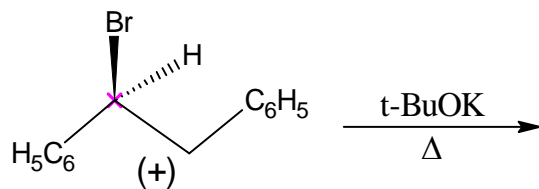


- (1) (I) < (III) < (II) (2) (II) < (III) < (I) (3) (III) < (II) < (I) (4) (II) < (I) < (III)

Key (4)

Sol. Rate of S_N1 depends on stability of carbocation formed

56. The major product obtained in the following reaction is:

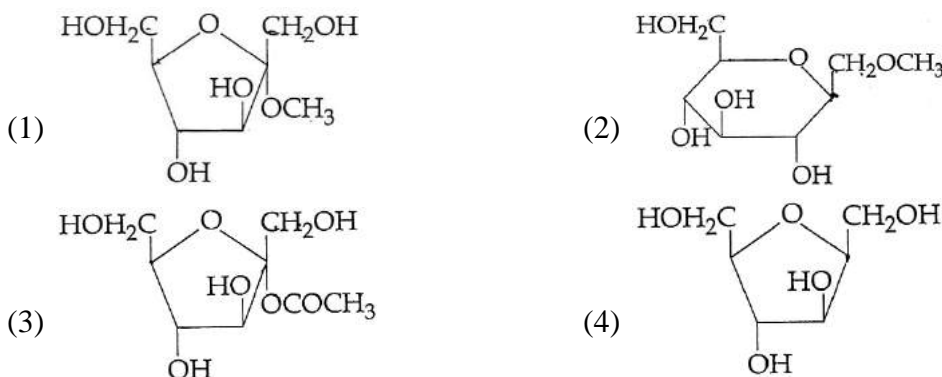


- (1) (+)C₆H₅CH(O^tBu)CH₂C₆H₅ (2) (-)C₆H₅CH(O^tBu)CH₂C₆H₅
 (3) (±)C₆H₅CH(O^tBu)CH₂C₆H₅ (4) C₆H₅CH=CHC₆H₅

Key (4)

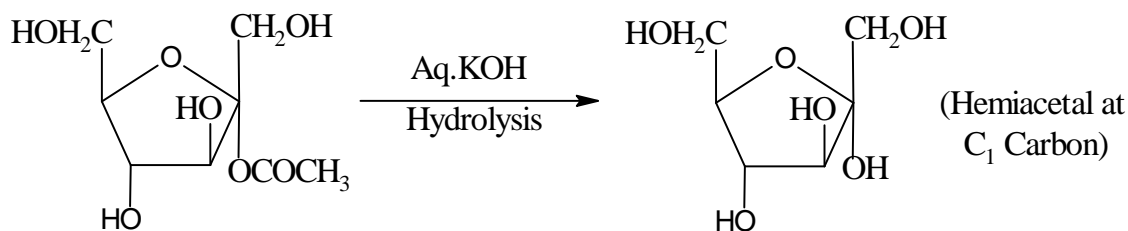
Sol. As t-BuOK with heat being very strong base results into elimination reaction to produce alkene

57. Which of the following compounds will behave as a reducing sugar in an aqueous KOH solution?



Key (3)

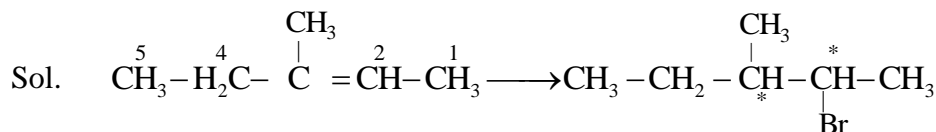
Sol. After hydrolysis in basic medium (Aq.KOH) only (3) option will produce hemiacetal which is a reducing sugar.



58. 3-Methyl-pent-2-ene on reaction with HBr in presence of peroxide forms an addition product. The number of possible stereoisomers for the product is:

- (1) Two (2) Four (3) Six (4) Zero

Key (2)

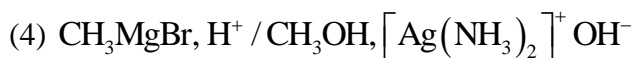
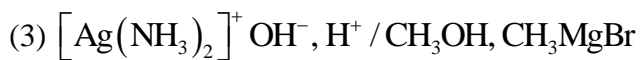
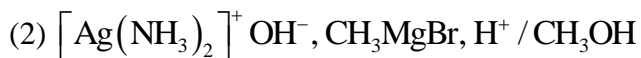
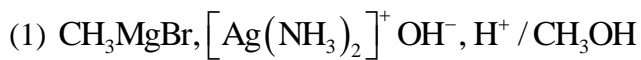
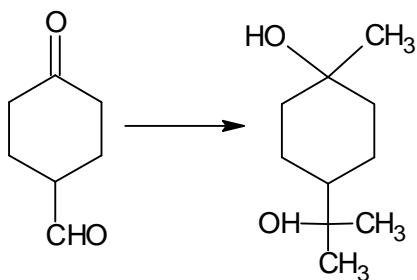


No. of stereoisomes = 2ⁿ

$$= 2^2$$

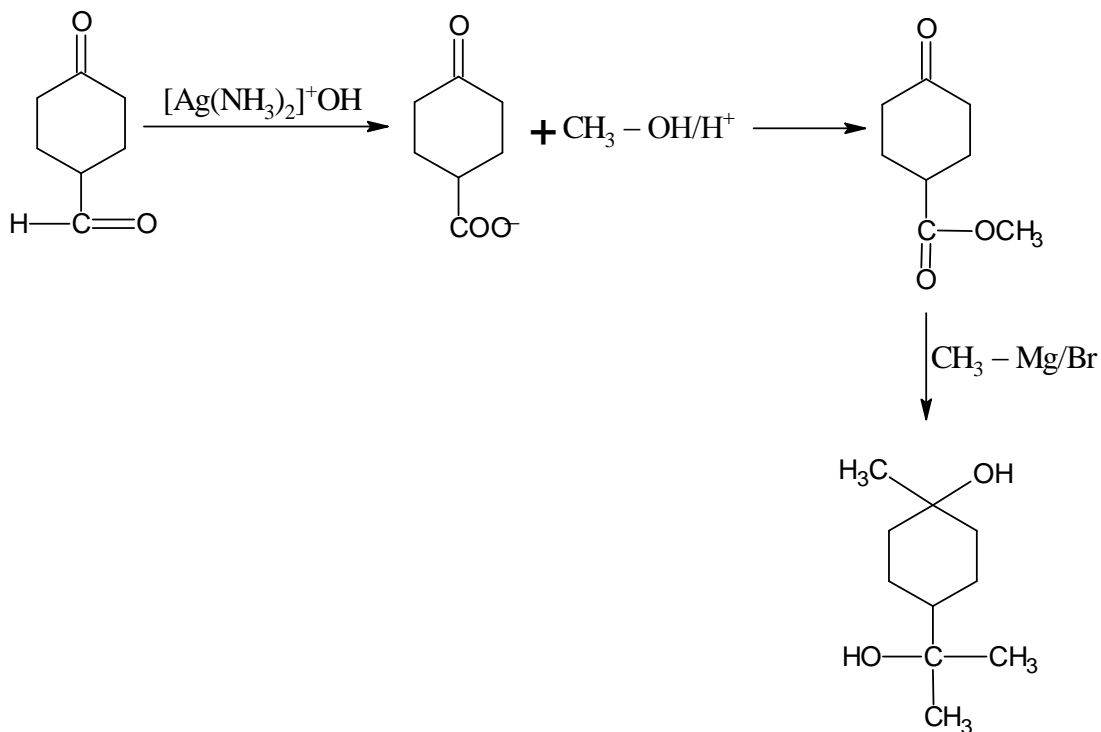
$$= 4$$

59. The correct sequence of reagents for the following conversion will be:

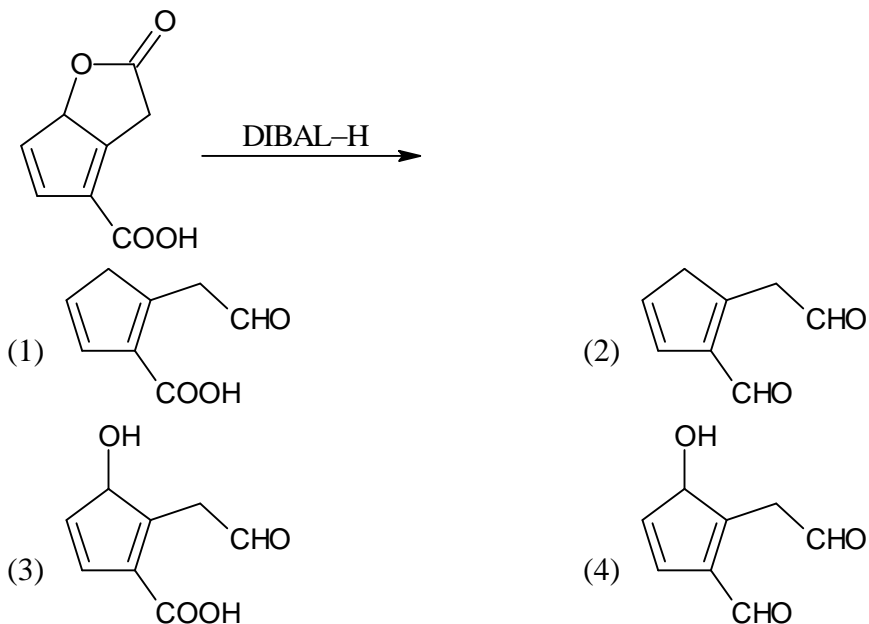


Key (3)

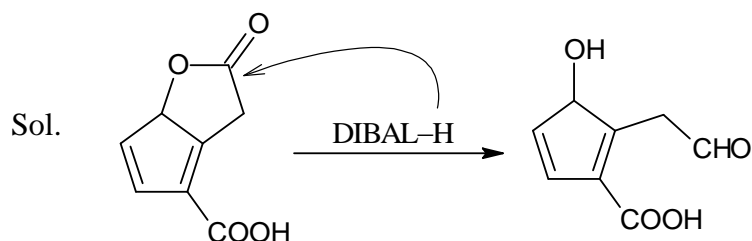
Sol.



60. The major product obtained in the following reaction is:



Key (3)



MATHEMATICS

61. The function $f : \mathbb{R} \rightarrow \left[-\frac{1}{2}, \frac{1}{2}\right]$ defined as $f(x) = \frac{x}{1+x^2}$, is:

- (1) injective but not surjective (2) surjective but not injective
 (3) neither injective nor surjective (4) invertible

Key. (2)

Sol. $f : \mathbb{R} \rightarrow \left[-\frac{1}{2}, \frac{1}{2}\right]$

$$f(x) = \frac{x}{1+x^2}$$

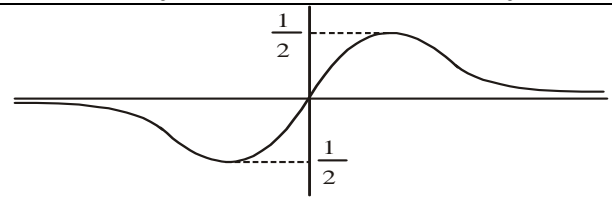
$$f'(x) = \frac{1-x^2}{(1+x^2)^2}$$

$\Rightarrow f(x)$ is many-one

$$f(-1) = \frac{-1}{2}, f(1) = \frac{1}{2}$$

$$\text{Range of } f(x) = \left[-\frac{1}{2}, \frac{1}{2}\right]$$

So $f(x)$ is many-one and onto.



62. If, for a positive integer n , the quadratic equation

$$x(x+1) + (x+1)(x+2) + \dots + (x+n-1)(x+n) = 10n$$

has two consecutive integral solutions, then n is equal to:

- (1) 9 (2) 10 (3) 11 (4) 12

Key. (3)

Sol. $x(x+1) + (x+1)(x+2) + (x+2)(x+3) + \dots + (x+(n-1))(x+n) = 10n$

$$\Rightarrow nx^2 + (0+1+1+2+2+3+\dots+(n-1)+n)x + (0.1+1.2+2.3+\dots+(n-1)n) = 10n$$

$$\Rightarrow nx^2 + n^2x + \frac{n(n^2-1)}{3} = 10n$$

$$\Rightarrow x^2 + nx + \frac{n^2-1}{3} = 10$$

$$\Rightarrow 3x^2 + 3nx + n^2 - 31 = 0$$

Roots are $\alpha, \alpha+1$

$$\Rightarrow \frac{\sqrt{D}}{|a|} = 1$$

$$\sqrt{9n^2 - 12(n^2 - 31)} = 3$$

$$9n^2 - 12(n^2 - 31) = 9$$

$$3n^2 - 4n^2 + 124 = 3$$

$$n^2 = 121$$

$$\Rightarrow n = 11$$

63. Let ω be a complex number such that $2\omega + 1 = z$ where $z = \sqrt{-3}$.

If $\begin{vmatrix} 1 & 1 & 1 \\ 1 & -\omega^2 - 1 & \omega^2 \\ 1 & \omega^2 & \omega^7 \end{vmatrix} = 3k$, then k is equal to:

- (1) z (2) -1 (3) 1 (4) -z

Key. (4)

Sol. $2\omega + 1 = \sqrt{-3} = \sqrt{3}i$

$$\omega = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$$

Which is cube root of unity

$$\begin{aligned} \begin{vmatrix} 1 & 1 & 1 \\ 1 & -\omega^2 - 1 & \omega^2 \\ 1 & \omega^2 & \omega^7 \end{vmatrix} &= \begin{vmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{vmatrix} \\ &= 1[\omega^2 - \omega^4] + 1[\omega^2 - \omega] + 1[\omega^2 - \omega] \\ &= \omega^2 - \omega + \omega^2 - \omega + \omega^2 - \omega \\ &= 3(\omega^2 - \omega) \\ &= 3(-1 - \omega - \omega) = -3(1 + 2\omega) = -3z \\ &\Rightarrow k = -z \end{aligned}$$

64. If $A = \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}$, then $\text{adj} (3A^2 + 12A)$ is equal to:

- (1) $\begin{bmatrix} 51 & 63 \\ 84 & 72 \end{bmatrix}$ (2) $\begin{bmatrix} 51 & 84 \\ 63 & 72 \end{bmatrix}$ (3) $\begin{bmatrix} 72 & -63 \\ -84 & 51 \end{bmatrix}$ (4) $\begin{bmatrix} 72 & -84 \\ -63 & 51 \end{bmatrix}$

Key. (1)

Sol. $A = \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix} \Rightarrow A^2 = \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix} \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 16 & -9 \\ -12 & 13 \end{bmatrix} \Rightarrow 3A^2 = \begin{bmatrix} 48 & -27 \\ -36 & 39 \end{bmatrix}$$

$$12A = \begin{bmatrix} 24 & -36 \\ -48 & 12 \end{bmatrix}$$

$$3A^2 + 12A = \begin{bmatrix} 72 & -63 \\ -84 & 51 \end{bmatrix}$$

$$\text{Adj} (3A^2 + 12A) = \begin{bmatrix} 51 & 63 \\ 84 & 72 \end{bmatrix}$$

65. If S is the set of distinct values of ‘b’ for which the following system of linear equations:

$$x + y + z = 1$$

$$x + ay + z = 1$$

$$ax + by + z = 0$$

has no solution, then S is:

- (1) an infinite set
- (2) a finite set containing two or more elements
- (3) a singleton
- (4) an empty set

Key. (3)

Sol. $x + y + z = 1$

$$x + ay + z = 1$$

$$ax + by + z = 0$$

Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & 1 \\ a & b & 1 \end{bmatrix}$

$$\det(A) = a - b + a - 1 + b - a^2 = -(a - 1)^2$$

For No solution $\det(A) = 0 \Rightarrow a = 1$

Now system become

$$x + y + z = 1$$

$$x + y + z = 1$$

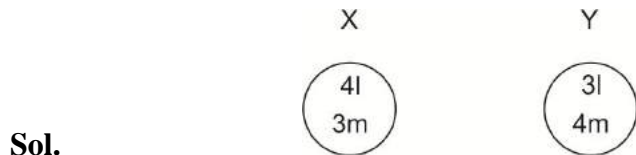
$$x + by + z = 0$$

For No solution $b = 1$

66. A man X has 7 friends, 4 of them are ladies and 3 are men. His wife Y also has 7 friends, 3 of them are ladies and 4 are men. Assume X and Y have no common friends. Then the total number of ways in which X and Y together can throw a party inviting 3 ladies and 3 men, so that 3 friends of each of X and Y are in this party, is:

- (1) 468 (2) 469 (3) 484 (4) 485

Key. (4)



CASE - 1	3 - l	-	3 m
CASE- 2	2l, 1m	-	1l, 2m
CASE - 3	1l, 2m	-	2l, 1m
CASE -4	3m	→	3l.

From all the cases

$$\begin{aligned} \text{Total no. of ways} &= {}^4C_3 \times {}^4C_3 + {}^4C_2 \times {}^3C_1 \times {}^3C_1 \times {}^4C_2 + {}^4C_1 \times {}^3C_2 \times {}^3C_2 \times {}^4C_1 + {}^3C_3 \times {}^3C_3 \\ &= 485 \end{aligned}$$

67. The value of

$$\left({}^{21}C_1 - {}^{10}C_1 \right) + \left({}^{21}C_2 - {}^{10}C_2 \right) + \left({}^{21}C_3 - {}^{10}C_3 \right) + \left({}^{21}C_4 - {}^{10}C_4 \right) + \dots + \left({}^{21}C_{10} - {}^{10}C_{10} \right) \text{ is:}$$

- (1) $2^{21} - 2^{10}$ (2) $2^{20} - 2^9$ (3) $2^{20} - 2^{10}$ (4) $2^{21} - 2^{11}$

Key. (3)

Sol. ${}^{21}C_1 + {}^{21}C_2 + \dots + {}^{21}C_{10} - \left({}^{10}C_1 + \dots + {}^{10}C_{10} \right) = \frac{2^{21} - 2}{2} - (2^{10} - 1) = 2^{20} - 2^{10}$

Sol. $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\cot x - \cos x}{(\pi - 2x)^3}$

Let $x = \frac{\pi}{2} + h$,

$$\lim_{h \rightarrow 0} \frac{-\tanh + \sinh}{(-8h)^3} = \lim_{h \rightarrow 0} \frac{(\cosh - 1)\sinh}{-8h^3} = \frac{1}{16}$$

71. If for $x \in \left(0, \frac{1}{4}\right)$, the derivative of $\tan^{-1}\left(\frac{6x\sqrt{x}}{1-9x^3}\right)$ is $\sqrt{x} \cdot g(x)$, then $g(x)$ equals:

- (1) $\frac{3x\sqrt{x}}{1-9x^3}$ (2) $\frac{3x}{1-9x^3}$ (3) $\frac{3}{1+9x^3}$ (4) $\frac{9}{1+9x^3}$

Key. (4)

Sol. $f(x) = \tan^{-1}\left(\frac{2(3x\sqrt{x})}{1-(3x\sqrt{x})^2}\right)$

Put $3x\sqrt{x} = \tan \theta$

$$= \tan^{-1}\left(\frac{2 \tan \theta}{1 - \tan^2 \theta}\right)$$

$$= \tan^{-1}(\tan 2\theta)$$

$$f(x) = 2\theta \quad \left(\text{Since } 0 < \theta < \frac{\pi}{4}\right)$$

$$f(x) = 2 \tan^{-1}(3x\sqrt{x})$$

$$f'(x) = \frac{2}{1+(3x\sqrt{x})^2} \cdot 3 \times \frac{3}{2} \cdot x^{1/2}$$

$$f'(x) = 9\sqrt{x} \cdot \left(\frac{1}{1+9x^3}\right)$$

$$g(x) = \frac{9}{1+9x^3}$$

72. The normal to the curve $y(x - 2)(x - 3) = x + 6$ at the point where the curve intersects the y-axis passes through the point:

- (1) $\left(\frac{1}{2}, \frac{1}{2}\right)$ (2) $\left(\frac{1}{2}, -\frac{1}{3}\right)$ (3) $\left(\frac{1}{2}, \frac{1}{3}\right)$ (4) $\left(-\frac{1}{2}, \frac{1}{3}\right)$

Key. (1)

Sol. $y(x - 2)(x - 3) = x + 6$... (i)

Curve cuts the y-axis when $x = 0$

Hence (i) $\Rightarrow y(-2)(-3) = 6$

$$6y = 6$$

$$y = 1$$

Differentiate (i) w.r.t. x

$$\frac{dy}{dx}(x-2)(x-3) + y(x-3) + y(x-2) = 1$$

At (0, 1), $\frac{dy}{dx} = +1$

Slope of normal = -1

Equation of normal at (0, 1) is

$$y - 1 = -1(x - 0)$$

$$y = -x + 1$$

This passes through $\left(\frac{1}{2}, \frac{1}{2}\right)$

73. Twenty meters of wire is available for fencing off a flower-bed in the form of a circular sector. Then the maximum area (in sq. m) of the flower-bed, is:
 (1) 10 (2) 25 (3) 30 (4) 12.5

Key. (2)

Sol. Length of wire fencing circular sector = 20

Hence $\ell + r + r = 20$

$$\ell + 2r = 20$$

$$\text{Area of flower bed} = \left(\frac{\theta}{2\pi}\right) \times \pi r^2$$

$$= \frac{\ell \cdot \pi r^2}{r 2\pi}$$

$$= \frac{\ell r}{2} = \frac{r(20 - 2r)}{2}$$

$$= r(10 - r)$$

$$f(r) = 10r - r^2$$

$$f'(r) = 10 - 2r$$

For maxima or minima $f'(r) = 0$

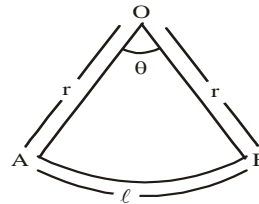
$$\Rightarrow r = 5$$

For $r = 5$

$$f''(r) < 0$$

Hence area will be maximum when $r = 5$.

$$\text{Maximum area} = 10(5) - (5)^2 = 50 - 25 = 25$$



74. Let $I_n = \int \tan^n x \, dx$, ($n > 1$). If $I_4 + I_6 = a \tan^5 x + bx^5 + C$, where C is a constant of integration, then the ordered pair (a, b) is equal to:

- (1) $\left(\frac{1}{5}, 0\right)$ (2) $\left(\frac{1}{5}, -1\right)$ (3) $\left(-\frac{1}{5}, 0\right)$ (4) $\left(-\frac{1}{5}, 1\right)$

Key. (1)

Sol. $I_n = \int \tan^n x \, dx$ ($n > 1$)

$$I_{n+2} = \int \tan^{n+2} x \, dx$$

$$I_n + I_{n+2} = \int \tan^n x (1 + \tan^2 x) \, dx$$

$$= \int \tan^n x \sec^2 x \, dx$$

$$= \frac{\tan^{n+1} x}{n+1} + c$$

$n = 4$

$$I_4 + I_6 = \frac{\tan^5 x}{5} + c$$

$a = \frac{1}{5}, b = 0$

The ordered pair $\left(\frac{1}{5}, 0\right)$

75. The integral $\int_{\pi/4}^{3\pi/4} \frac{dx}{1 + \cos x}$ is equal to:

- (1) 2 (2) 4 (3) -1 (4) -2

Key. (1)

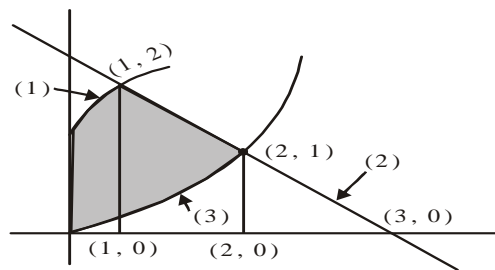
Sol. $I = \int_{\pi/4}^{3\pi/4} \frac{dx}{1 + \cos x}$
 $= \int_{\pi/4}^{3\pi/4} \frac{dx}{2 \cos^2 \frac{x}{2}} = \frac{1}{2} \int_{\pi/4}^{3\pi/4} \sec^2 \frac{x}{2} dx$
 $= \tan \frac{x}{2} \Big|_{\pi/4}^{3\pi/4}$
 $= \tan \frac{3\pi}{8} - \tan \frac{\pi}{8} = \cot \frac{\pi}{8} - \tan \frac{\pi}{8}$
 $= 2 \cot \frac{\pi}{4} = 2$

76. The area (in sq. units) of the region $\{(x, y) : x \geq 0, x + y \leq 3, x^2 \leq 4y \text{ and } y \leq 1 + \sqrt{x}\}$ is:

- (1) $\frac{3}{2}$ (2) $\frac{7}{3}$ (3) $\frac{5}{2}$ (4) $\frac{59}{12}$

Key. (3)

Sol.



- (1) $y = 1 + \sqrt{x}$
 (2) $x + y = 3$
 (3) $x^2 = 4y$

Required area = $\int_0^1 (1 + \sqrt{x}) dx + \int_1^2 (3 - x) dx - \int_0^2 \frac{x^2}{4} dx$
 $= \left(1 + \frac{2}{3}\right) + 3 - \frac{3}{2} - \frac{8}{12}$
 $= \frac{5}{3} + \frac{3}{2} - \frac{2}{3} = \frac{5}{2}$

77. If $(2 + \sin x) \frac{dy}{dx} + (y + 1) \cos x = 0$ and $y(0) = 1$, then $y\left(\frac{\pi}{2}\right)$ is equal to:

- (1) $-\frac{2}{3}$ (2) $-\frac{1}{3}$ (3) $\frac{4}{3}$ (4) $\frac{1}{3}$

Key. (4)

Sol. $(2 + \sin x) \frac{dy}{dx} + (y + 1) \cos x = 0$

$$\frac{d}{dx} \{(2 + \sin x)(y + 1)\} = 0$$

$$(2 + \sin x)(y + 1) = k \quad \dots (i)$$

$$y(0) = 1$$

$$\Rightarrow (2 + 0)(1 + 1) = k$$

$$\Rightarrow k = 4$$

$$(i) \Rightarrow (2 + \sin x)(y + 1) = 4$$

$$\text{When } x = \frac{\pi}{2}$$

$$(2 + 1)(y + 1) = 4$$

$$y + 1 = \frac{4}{3}$$

$$y = \frac{1}{3}$$

78. Let k be an integer such that the triangle with vertices $(k, -3k)$, $(5, k)$ and $(-k, 2)$ has area 28 sq. units. Then the orthocenter of this triangle is at the point:

- (1) $\left(1, \frac{3}{4}\right)$ (2) $\left(1, -\frac{3}{4}\right)$ (3) $\left(2, \frac{1}{2}\right)$ (4) $\left(2, -\frac{1}{2}\right)$

Key. (3)

Sol. Area of triangle = 28

$$\begin{vmatrix} k & -3k & 1 \\ 5 & k & 1 \\ -k & 2 & 1 \end{vmatrix} = \pm 56$$

$$k(k - 2) + 3k(5 + k) + 1(10 + k^2) = \pm 56$$

$$5k^2 + 13k + 10 = \pm 56$$

$$5k^2 + 13k + 66 = 0 \quad \text{OR} \quad 5k^2 + 13k - 46 = 0$$

$$\text{Not possible} \quad 5k^2 + 23k - 10k - 46 = 0$$

$$k = 2, k = -\frac{23}{5}$$

$k = 2$ (Since k is integer)

Vertices of triangle are $(2, -6)$, $(5, 2)$ and $(-2, 2)$

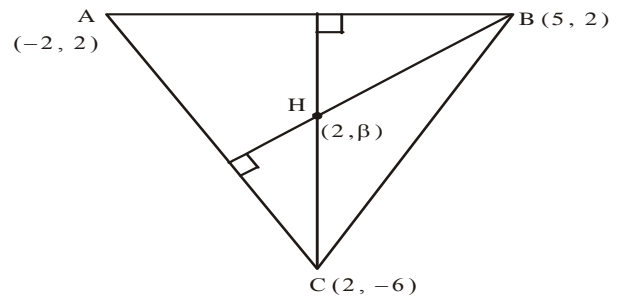
$$(\text{Slope of BH})(\text{Slope of AC}) = -1$$

$$\frac{2-\beta}{5-2} \times \frac{2+6}{-2-2} = -1$$

$$4-2\beta = 3$$

$$\beta = \frac{1}{2}$$

Ortho centre i.e. $\left(2, \frac{1}{2}\right)$

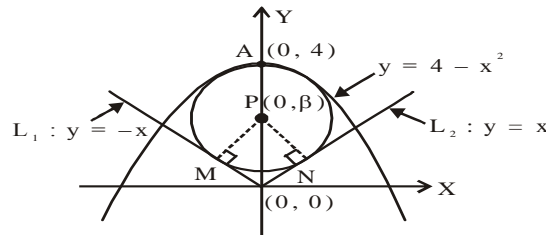


79. The radius of a circle, having minimum area, which touches the curve $y = 4 - x^2$ and the lines, $y = |x|$ is:

- (1) $2(\sqrt{2}-1)$ (2) $4(\sqrt{2}-1)$ (3) $4(\sqrt{2}+1)$ (4) $2(\sqrt{2}+1)$

Key. (2)

Sol.



Let $P \equiv (0, \beta)$ be the centre, then since the circle touches the lines $y = |x|$.

Then, $PM \perp L_1$ and $PN \perp L_2$

$$PM = \frac{\beta}{\sqrt{2}} = PA$$

$$\Rightarrow \frac{\beta}{\sqrt{2}} = 4 - \beta \Rightarrow \beta = \frac{4\sqrt{2}}{1+\sqrt{2}}$$

Hence radius is $4 - \beta = 4(\sqrt{2}-1)$

80. The eccentricity of an ellipse whose centre is at the origin is $\frac{1}{2}$. If one of its directrices is $x = -4$,

then the equation of the normal to it at $\left(1, \frac{3}{2}\right)$ is:

- (1) $4x - 2y = 1$ (2) $4x + 2y = 7$ (3) $x + 2y = 4$ (4) $2y - x = 2$

Key. (1)

Sol. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$... (i)

$$e = \frac{1}{2}$$

$$x = \frac{a}{e} \Rightarrow a = -2$$

Put value of a in (i)

We get $b^2 = 3$

$$\Rightarrow \frac{x^2}{4} + \frac{y^2}{3} = 1$$

So, normal at $\left(1, \frac{3}{2}\right)$

$$\frac{4x}{1} - \frac{3y}{3/2} = 4 - 3$$

$$\Rightarrow 4x - 2y = 1$$

81. A hyperbola passes through the point $P(\sqrt{2}, \sqrt{3})$ and has foci at $(\pm 2, 0)$. Then the tangent to this hyperbola at P also passes through the point:

- (1) $(2\sqrt{2}, 3\sqrt{3})$ (2) $(\sqrt{3}, \sqrt{2})$ (3) $(-\sqrt{2}, -\sqrt{3})$ (4) $(3\sqrt{2}, 2\sqrt{3})$

Key (1)

Sol. Focus $= (\pm ae, 0) = (\pm 2, 0) \Rightarrow ae = 2$

$$b^2 = a^2 e^2 - a^2 = 4 - a^2$$

Equation of hyperbola is $\frac{x^2}{a^2} - \frac{y^2}{4-a^2} = 1$

Hyperbola passes through $P(\sqrt{2}, \sqrt{3})$

$$\Rightarrow \frac{2}{a^2} - \frac{3}{4-a^2} = 1 \Rightarrow a^2 = 1 \text{ or } 8 \text{ (8 is rejected because 'e' is not less than 1)}$$

$$\Rightarrow b^2 = a^2 e^2 - a^2 = 3$$

Equation of hyperbola is $\frac{x^2}{1} - \frac{y^2}{3} = 1$

Tangent at $P(\sqrt{2}, \sqrt{3})$ is

$$\frac{xx'}{1} - \frac{yy'}{3} = 1 \Rightarrow \frac{\sqrt{2}x}{1} - \frac{\sqrt{3}y}{3} = 1 \text{ (Satisfying only option 1)}$$

82. The distance of the point $(1, 3, -7)$ from the plane passing through the point $(1, -1, -1)$, having normal perpendicular to both the lines $\frac{x-1}{1} = \frac{y+2}{-2} = \frac{z-4}{3}$ and $\frac{x-2}{2} = \frac{y+1}{-1} = \frac{z+7}{-1}$, is:

(1) $\frac{10}{\sqrt{83}}$

(2) $\frac{5}{\sqrt{83}}$

(3) $\frac{10}{\sqrt{74}}$

(4) $\frac{20}{\sqrt{74}}$

Key (1)

Sol. Equation of plane passing through the point $(1, -1, -1)$, having normal perpendicular to lines

$$\frac{x-1}{1} = \frac{y+2}{-2} = \frac{z-4}{3} \text{ \& } \frac{x-2}{2} = \frac{y+1}{-1} = \frac{z+7}{-1} \text{ is } \begin{vmatrix} x-1 & y+1 & z+1 \\ 1 & -2 & 3 \\ 2 & -1 & -1 \end{vmatrix} = 0$$

$$\Rightarrow 5x + 7y + 3z + 5 = 0 \quad \dots(i)$$

Now distance of the point $(1, 3, -7)$ from the plane (i) is $\left| \frac{5(1) + 7(3) + 3(-7) + 5}{\sqrt{5^2 + 7^2 + 3^2}} \right| = \frac{10}{\sqrt{83}}$

83. If the image of the point $P(1, -2, 3)$ in the plane $2x + 3y - 4z + 22 = 0$ measured parallel to the line

$\frac{x}{1} = \frac{y}{4} = \frac{z}{5}$ is Q, then PQ is equal to:

(1) $2\sqrt{42}$

(2) $\sqrt{42}$

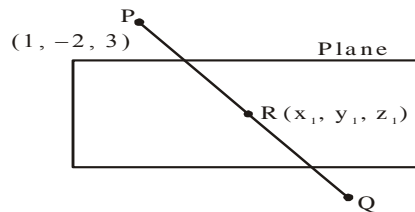
(3) $6\sqrt{5}$

(4) $3\sqrt{5}$

Key (1)

Sol. Let $R(x_1, y_1, z_1)$ be the point of intersection of line PQ and the given plane, then

$$\frac{x_1-1}{1} = \frac{y_1+2}{4} = \frac{z_1-3}{5} = \lambda$$



$$x_1 = \lambda + 1, y_1 = 4\lambda - 2, z_1 = 5\lambda + 3$$

R satisfies the equation of given plane, then

$$2\lambda + 2 + 12\lambda - 6 - 20\lambda - 12 + 22 = 0$$

$$-6\lambda + 6 = 0 \Rightarrow \lambda = 1$$

$$\therefore PR = \sqrt{\lambda^2 + (4\lambda)^2 + (5\lambda)^2} = \sqrt{42\lambda^2} = \sqrt{42}$$

$$PQ = 2PR = 2\sqrt{42}$$

$q =$ probability of non-occurrence of an event

Hence, variance of the number of green balls drawn is $= npq = 10 \times \frac{3}{5} \times \frac{2}{5} = \frac{12}{5}$

86. For three events A, B and C, $P(\text{Exactly one of A or B occurs}) = P(\text{Exactly one of B or C occurs}) = P(\text{Exactly one of C or A occurs}) = \frac{1}{4}$ and $P(\text{All the three events occur simultaneously}) = \frac{1}{16}$

.Then the probability that at least one of the events occurs, is:

- (1) $\frac{7}{16}$ (2) $\frac{7}{64}$ (3) $\frac{3}{16}$ (4) $\frac{7}{32}$

Key (1)

Sol. $P(\text{Exactly one of A or B occurs}) = \frac{1}{4} = P(A) + P(B) - 2P(A \cap B)$ (i)

$P(\text{Exactly one of B or C occurs}) = \frac{1}{4} = P(B) + P(C) - 2P(B \cap C)$ (ii)

$P(\text{Exactly one of C or A occurs}) = \frac{1}{4} = P(C) + P(A) - 2P(C \cap A)$ (iii)

Adding (i), (ii) and (iii)

$$2(P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A)) = \frac{3}{4}$$

$$\Rightarrow P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) = \frac{3}{8}$$

$P(\text{All the three events occurs simultaneously}) = P(A \cap B \cap C) = \frac{1}{16}$

Probability that at least one of the events occurs $= P(A \cup B \cup C)$

$$= P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$$

$$\Rightarrow P(A \cup B \cup C) = \frac{3}{8} + \frac{1}{16} = \frac{6+1}{16} = \frac{7}{16}$$

87. If two different numbers are taken from the set $\{0,1,2,3,\dots,10\}$; then the probability that their sum as well as absolute difference are both multiples of 4, is:

- (1) $\frac{12}{55}$ (2) $\frac{14}{45}$ (3) $\frac{7}{55}$ (4) $\frac{6}{55}$

Key (4)

Sol. $a + b = 4k$

$|a - b| = 4m$, where $k, m \in \mathbb{Z}$.

Hence, a and b must be chosen either from $\{0, 4, 8\}$ or $\{2, 6, 10\}$

$$\therefore \text{Probability} = \frac{{}^3C_2 + {}^3C_2}{{}^{11}C_2} = \frac{6}{55}$$

88. If $5(\tan^2 x - \cos^2 x) = 2\cos 2x + 9$, then the value of $\cos 4x$ is:

- (1) $\frac{1}{3}$ (2) $\frac{2}{9}$ (3) $-\frac{7}{9}$ (4) $-\frac{3}{5}$

Key (3)

Sol. $5(\tan^2 x - \cos^2 x) = 2\cos 2x + 9$

$$\frac{5(1 - \cos^2 x - \cos^4 x)}{\cos^2 x} = 2(2\cos^2 x - 1) + 9$$

$$5 - 5\cos^2 x - 5\cos^4 x = 4\cos^4 x - 2\cos^2 x + 9\cos^2 x$$

$$9\cos^4 x + 12\cos^2 x - 5 = 0$$

$$\cos^2 x = \frac{-12 \pm \sqrt{144 + 180}}{18}$$

$$\Rightarrow \cos^2 x = \frac{1}{3} \quad (\because \cos^2 x = \frac{-5}{3} \text{ is neglected})$$

Now $\cos 4x = 2\cos^2 2x - 1$

$$= 2(2\cos^2 x - 1)^2 - 1$$

$$= 8\cos^4 x - 8\cos^2 x + 1$$

$$= \frac{8}{9} - \frac{8}{3} + 1$$

$$= \frac{8 - 24 + 9}{9} = \frac{-7}{9}$$

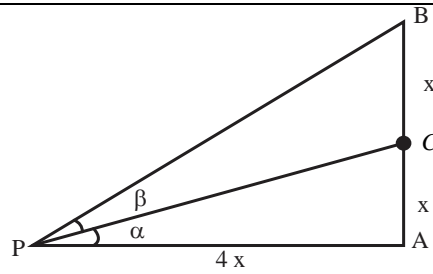
89. Let a vertical tower AB have its end A on the level ground. Let C be the mid-point of AB and P be a point on the ground such that $AP = 2 AB$. If $\angle BPC = \beta$, then $\tan \beta$ is equal to:

- (1) $\frac{1}{4}$ (2) $\frac{2}{9}$ (3) $\frac{4}{9}$ (4) $\frac{6}{7}$

Key (2)

Sol. $\tan \alpha = \frac{1}{4}$

$$\tan(\alpha + \beta) = \frac{1}{2}$$



$$\frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta} = \frac{1}{2}$$

$$\frac{1}{4} + \tan \beta = \left(1 - \frac{\tan \beta}{4}\right) \frac{1}{2}$$

$$\frac{1}{2} + 2 \tan \beta = 1 - \frac{\tan \beta}{4}$$

$$\frac{1}{2} = \frac{9 \tan \beta}{4}, \tan \beta = \frac{2}{9}$$

90. The following statement $(p \rightarrow q) \rightarrow [(\sim p \rightarrow q) \rightarrow q]$ is:

- (1) equivalent to $\sim p \rightarrow q$
- (2) equivalent $p \rightarrow \sim q$
- (3) a fallacy
- (4) a tautology

Key (4)

Sol.

P	q	$\sim p$	$(p \rightarrow q)$	$(\sim p \rightarrow q)$	$[(\sim p \rightarrow q) \rightarrow q]$	$(p \rightarrow q) \rightarrow [(\sim p \rightarrow q) \rightarrow q]$
T	T	F	T	T	T	T
T	F	F	F	T	F	T
F	T	T	T	T	T	T
F	F	T	T	F	T	T

JEE (Main)-2017 Analysis

Chemistry Analysis			
UNIT & TOPIC NAME	No. of Question	Total Marks	% Weightage
Inorganic Chemistry			
Chemical Bonding	1	4	3.3
d-block & f-block			
Metallurgy			
p-block	3	12	9.9
Periodic Table & Periodicity in Properties	1	4	3.3
Qualitative Analysis			
s-block	1	4	3.3
Transition Elements and Coordination Chemistry	1	4	3.3
Organic Chemistry			
Alkyl Halide, Alcohol & Ether	2	8	6.6
Aromatic Compound			
Carbohydrates, Amino Acids and Polymers	3	12	9.9
IUPAC & Isomerism	1	4	3.3
Practical Organic Chemistry			
Physical Chemistry			
Atomic Structure			
Chemical Equilibrium	1	4	3.3
Chemical Kinetics			
Electrochemistry	1	4	3.3
Mole Concept	2	8	6.6
Solid State	1	4	3.3
Solution & Colligative Properties	1	4	3.3
Surface Chemistry	1	4	3.3
Thermodynamics & Thermochemistry	2	8	6.6
GRAND TOTAL	30	120	

Physics Analysis			
UNIT & TOPIC NAME	No. of Question	Total Marks	% Weight age
Mechanics	8	32	26.4
Heat and thermodynamics	4	16	13.2
Oscillations & waves	1	4	3.3
Electro magnetism	8	32	26.4
Optics	3	12	9.9
Modern physics	6	24	19.8
Grand Total	30	120	

Math Analysis			
UNIT & TOPIC NAME	No. of Question	Total Marks	% Weightage
Area under curve	1	4	3.3
Area Between two Curves			
Circle	1	4	3.3
Position of Two Circles			
Complex Numbers	1	4	3.3
Properties of Modulus			
Differential equation	1	4	3.3
Linear Differential Equation			
Ellipse	1	4	3.3
Tangents			
Parabola	1	4	3.3
Parametric Coordinates			
Probability	2	8	6.6
Classical Probability			
Quadratic Equation	1	4	3.3
Relation Between Roots & Coefficients			
Matrix & Determinants	2	8	6.6
Multiplication of Matrices			
Cramer's Rule			
Permutation & Combination	1	4	3.3

Fundamental Theorem of Counting			
Binomial Theorem	1	4	3.3
Summation of Series	1	4	3.3
Sequence & Series	1	4	3.3
Geometric Mean			
Miscellaneous Series			
Vector	1	4	3.3
Vector Triple Product			
Statistics	1	4	3.3
Mean			
Mathematical Reasoning	1	4	3.3
Negation			
Vector & -D	2	8	6.6
Family of Planes			
Intersection of Plane and Line			
Inverse Trigonometric function	1	4	3.3
Addition of Inverse Trigonometric Function			
Sets & Relation	1	4	3.3
Subset			
Limit of function	1	4	3.3
Standard Limit			
Continuity & Derivability			
Differentiability			
Tangents, Normal & Applications	2	8	6.6
Normal at a Point			
Application of Derivatives	1	4	3.3
Maxima & Minima			
Indefinite integration	1	4	3.3
Substitution			
Definite integration	1	4	3.3
Properties of DI			
Straight Line	1	4	3.3
Position of Point			
Family of Planes			
Height & distance	1	4	3.3
Angle of Elevation			
GRAND TOTAL	30	120	